SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

# JOURNAL OF MARINE RESEARCH

VOLUME VI, 1947

NUMBER 3

ISSUED DECEMBER 15, 1947, NEW HAVEN, CONN., U. S. A.

# JOURNAL OF MARINE RESEARCH

PUBLISHED BY

SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

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The Journal is normally issued in three numbers per year constituting one volume, except for 1942 through 1945, when only one volume was published. Volume VI will be completed in 1947. All the income derived from subscriptions will be used solely to increase the volume and improve the quality of the Journal for the benefit of its subscribers and contributors.

**Subscription price three dollars per volume, or one dollar per number. Address all communications to the Managing Editor. See inside rear cover for Note to Authors.**

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JOURNAL OF
MARINE RESEARCH

VOLUME VI 1947 NUMBER 3

# ENVIRONMENTAL CHARACTERISTICS OF A RIVER ESTUARY[1]

By

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*Washington College, Maryland*

## DESCRIPTION OF THE REGION

The Patuxent River is one of the larger tributaries of the Chesapeake Bay. It has its origin in the north-central part of Maryland at an altitude of about 800 feet and empties into the western side of the bay at a point about halfway between the mouth and upper limits of the bay. Its basin is a hilly, rolling country without lakes and is characterized by low shores that rise gently toward the interior. The soil consists of sand and clay and contains some limestone in local areas. The drainage area of the river is about 960 square miles, and its length measured on a straight line is about 80 miles (Bonsteel and Burke, 1907).

Consideration has been given to the lower and estuarine half of the river and to the region of the Chesapeake Bay opposite its mouth. The lower half of the river is estuarine in that its water is partly derived from the Chesapeake Bay. There exists in the lower half of the estuary (lower quarter of the river) a deep channel ranging in depth from 9 to 35 meters, which, except for an increase in depth a few miles inside the mouth of the estuary, generally decreases upstream. In the upper half of the estuary the channel varies in depth from 3 to 14 meters. The bottom of the estuary is generally muddy, the depth of

[1] Contribution No. 64 of the Chesapeake Biological Laboratory.

the mud decreasing upstream, except in the upper regions where it is sandy, as are the shores.

The tidal amplitude is 1.4 feet in the mouth of the estuary, increasing to 2.9 feet near its upper limits. Due to the gradual narrowing and reduction in depth of the estuary upstream, the current velocity increases in this direction from about 0.3 meters per second at full strength in the estuary mouth to about 0.5 meters per second near its upper limits. At each end of the estuary there is little difference between flood and ebb strength velocity, but, throughout the greater part of the estuary, flood strength velocity exceeds ebb strength velocity by about 0.2 meters per second (data from Haight, Finnegan and Anderson, 1930).

## METHODS OF STUDY

Twenty-one permanent stations, established in the estuarine lower half of the river and in the region of the Chesapeake Bay opposite its mouth (Fig. 41, exclusive of Stations 6 and 6A), were visited during 1936–1939 and 1944–1946 for studies of depth of light penetration and (for both surface and bottom waters) of temperature, salinity, and silicate, dissolved oxygen and hydrogen-ion concentrations. A total of 252 individual station visits were made, and sampling was sufficiently frequent to obtain data at representative stations for each of the twelve months of the year. Nitrates and nitrites were studied during 1936–1937, and plankton during 1936–1937 and the summer of 1944, the stations being visited on an average of five times each during 1936–1937.

A constant recording thermograph for both air and surface water was installed on the end of the pier at Solomons (Station 6), which extends 740 feet into the Patuxent River. The depth of the water at this station is two meters. The air thermometer is situated in the shade under the end of the pier about five feet above mean river level, and the surface water thermometer is beneath the air thermometer at a depth of about four feet below mean river level. A marigraph, for a continuous record of the water level, and a tide level gauge were installed at this station by the Coast and Geodetic Survey; hydrometers provided for density determinations of the water. These instruments are operated by the Chesapeake Biological Laboratory and the daily records are analyzed by the Coast and Geodetic Survey. Salinity and temperature data obtained for the 98-month period, November, 1937 to January, 1946 are used in this report. Also at Station 6, the dissolved oxygen concentration of the surface water was determined during 1938–1940 with a total of 353 determinations, the pH of the surface water during 1938–1943 with a total of 206 determinations, and

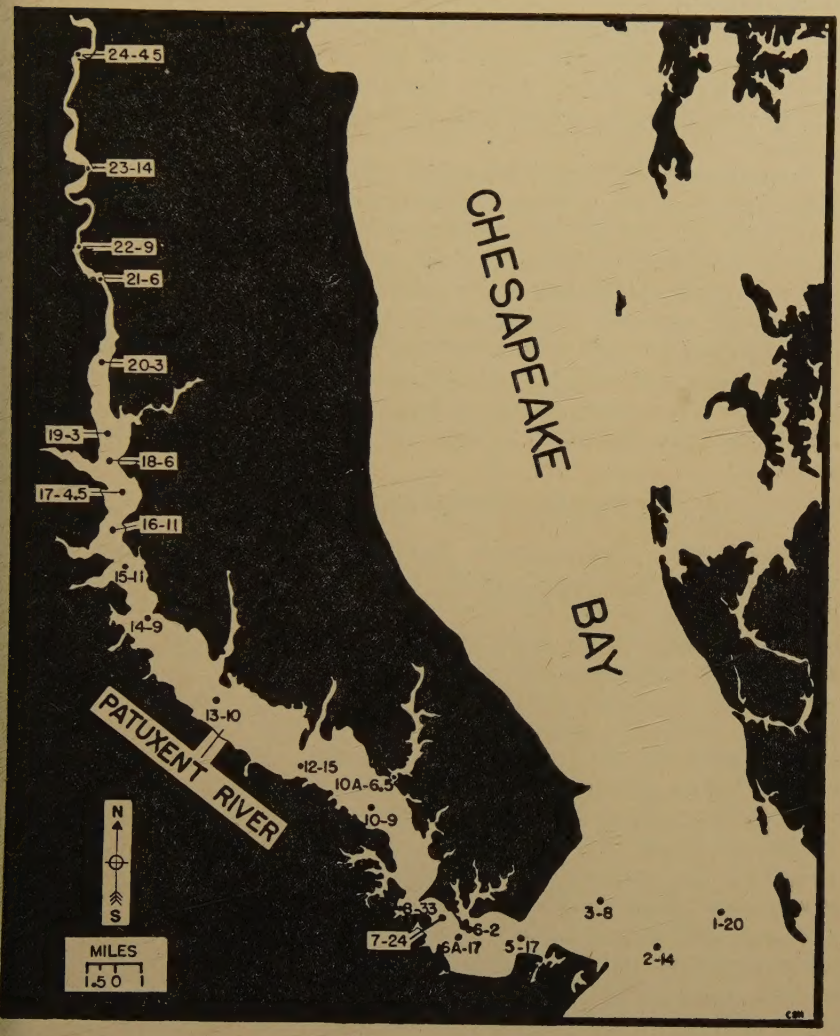


Figure 41. Locations and depths (meters) of sampling stations in the lower half of the Patuxent River and the Chesapeake Bay.

depth of light penetration during 1939–1943 with a total of 164 determinations.

In October, 1942, Station 6A was established and visited at intervals averaging nine days for a total of 155 visits through August, 1946. Both surface and bottom waters were studied for temperature, salinity and dissolved oxygen concentration, the depth at this station being 17 meters.

Samples were collected with either a Foerst or a Nansen bottle. As preservatives, 5 ml. of filtered neutralized 10% formaldehyde solution were used per 200 ml. of plankton sample, and 0.2 ml. of saturated mercurous chloride solution or 3 ml. of carbon disulfide were used per 100 ml. of silicate, nitrate and nitrite samples. Subsurface temperatures were recorded with reversing thermometers. The method of analysis used for each factor is stated in the discussion of the results.

Collections and analyses of samples were executed by several members of the Chesapeake Biological Laboratory in addition to the author, and gratitude is expressed to them for their large contribution to the work.

## ENVIRONMENTAL FACTORS

*Temperature.* Temperatures fluctuate daily and seasonal trends vary somewhat from year to year. For the study of seasonal variations in temperature, Station 6 is used for air and surface water and Station 6A for bottom water. The distance between the air and surface water thermometers is nine feet. The air is warmer than the surface water from the latter part of January until the latter part of June (Fig. 42, Table V). During the rest of the year the surface water is warmer than the air and has a year around average temperature of 0.5° C. higher than the air. The higher temperature of the surface water is the result of its receiving more heat from solar radiation than is received by the air.

The air temperature data at Station 6 were compared with the air temperature data of the same period at the U. S. Weather Bureau station at Solomons for a determination of any effect that proximity to water may have on the temperature of the air. This Weather Bureau station, operated by the Chesapeake Biological Laboratory, is located about 170 feet inland and about 910 feet from Station 6. Greater proximity to the water had no effect on the average yearly temperature of the air, there being no observable difference between the average yearly air temperatures at the two stations.

The temperatures of the surface and bottom waters both begin to decrease in early August (Fig. 42, Table V), but, because of the more rapid cooling of the surface water, the bottom water is generally

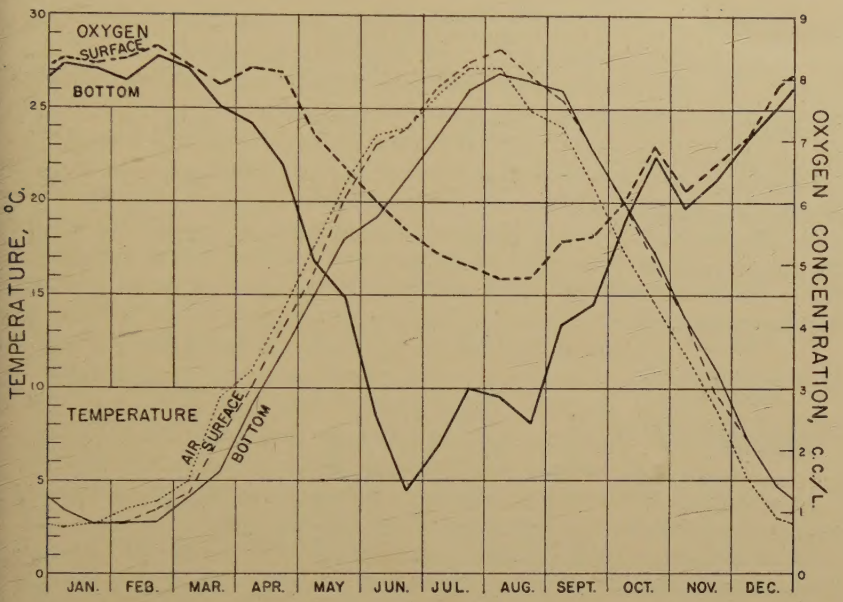


**Figure 42.** Seasonal variations at Station 6 and 6A in air, surface, and bottom temperatures and in surface and bottom dissolved oxygen concentrations.

warmer than the surface water from September to December, resulting in an overturn during this period. In December and January the bottom water reaches temperatures as low as those of the surface water, and is colder than the surface water from February to September. In January and February the temperature of the surface water generally fluctuates around the temperature of the point of its maximum density, resulting in convection currents, so that the overturn which begins in September does not thoroughly subside until March. The average surface-bottom density differential at Station 6A during the overturn from September through February is 0.0006, while during the rest of the year it is 0.0015. The surface-bottom density differential undergoes a decrease of 0.0006 between August and September at the time of the beginning of the overturn, and an increase of 0.0013 between February and March at the time of the beginning of the stratification period. Monthly density values at Station 6A are listed in Table V.

To manifest the longitudinal gradients in the physical and chemical factors within the estuary and between the estuary and the bay, the stations, other than 6 and 6A, are placed in four groups as follows:

TABLE I. Physical and Chemical Factors in Bay Group and in Groups 1, 2 and 3

| *Factor* | *Bay Group* | *Group 1* | *Group 2* | *Group 3* |
|---|---|---|---|---|
| Surface temperature range, °C. | 27.0 | 27.3 | 29.6 | 29.7 |
| Bottom temperature range, °C. | 25.4 | 25.9 | 26.4 | 28.9 |
| Surface temperature °C., March–July | 21.5 | 21.7 | 22.2 | 22.8 |
| Bottom temperature °C., March–July | 19.4 | 19.7 | 21.2 | 22.2 |
| Surface temperature °C., Sept.–Jan. | 19.0 | 18.7 | 18.3 | 17.7 |
| Bottom temperature °C., Sept.–Jan. | 19.4 | 19.0 | 18.5 | 17.7 |
| Surface-bottom salinity differential, % | 41 | 14 | 18 | 16 |
| Surface-bottom oxygen differential, c.c./l. | 2.3 | 1.9 | 1.6 | 1.0 |
| Depth of light penetration, cm. | 208 | 187 | 91 | 49 |
| Surface silicate conc. $10^{-3}$ mg. atoms/l. | 17 | 49 | 84 | 112 |
| Bottom silicate conc. $10^{-3}$ mg. atoms/l. | 14 | 46 | 73 | 110 |
| Surface silicate range $10^{-3}$ mg. atoms/l. | 4–31 | 8–83 | 16–200 | 55–312 |
| Bottom silicate range $10^{-3}$ mg. atoms/l. | 3–30 | 12–108 | 10–195 | 53–329 |
| Surface-bottom silicate differential, % | 21 | 5 | 15 | 4 |
| Surface nitrite conc. $10^{-3}$ mg. atoms/l. | 0.2 | 0.2 | 0.1 | 0.1 |
| Surface nitrite range $10^{-3}$ mg. atoms/l. | 0.0–1.3 | 0.0–1.4 | 0.0–0.4 | 0.0–0.4 |
| Bottom nitrite conc. $10^{-3}$ mg. atoms/l. | 0.3 | 0.3 | 0.2 | 0.1 |
| Bottom nitrite range $10^{-3}$ mg. atoms/l. | 0.0–1.5 | 0.0–1.5 | 0.0–1.6 | 0.0–0.4 |
| Surface nitrate conc. $10^{-3}$ mg. atoms/l. | 0.8 | 1.0 | 1.7 | 1.9 |
| Surface nitrate range $10^{-3}$ mg. atoms/l. | 0.0–2.5 | 0.0–3.5 | 0.0–5.0 | 0.0–4.8 |
| Bottom nitrate conc. $10^{-3}$ mg. atoms/l. | 1.4 | 1.7 | 1.9 | 2.0 |
| Bottom nitrate range $10^{-3}$ mg. atoms/l. | 0.0–4.7 | 0.4–4.8 | 0.0–5.8 | 0.0–8.2 |

Bay Group, Stations 1–3; Group 1, Stations 5–12; Group 2, Stations 13–18; and Group 3, Stations 19–24. The temperature ranges of both surface and bottom waters increase upstream (Table I). This is caused by the increasing remoteness of the stabilizing effect of the bay water and by the increasing surface-volume ratio of the water in this direction. A greater surface-volume ratio increases the heating effect of solar radiation in the summer and the cooling effect of evaporation in the winter. The temperature range of the surface water is greater than that of the bottom water in all groups, since in the summer the surface water receives more radiation than the bottom water, and in the winter the bottom water generally reaches a temperature no lower than the minimum temperature of the surface water.

Seasonal changes in temperature occur sooner in the upstream direction in both surface and bottom water. This is shown in Table I, where are listed the average temperatures of each of the four groups of stations for trips in which at least one station in each group was visited (1) during the period of increasing temperature, March through July, and (2) during the period of decreasing temperature, September through January. The average temperature increases upstream during the period of increasing temperature, while the average temperature decreases upstream during the period of decreasing temperature. That seasonal temperature changes are initiated sooner upstream is primarily the result of the increasing surface-volume ratio in this direction. In the case of the bottom water it is also due to decreasing depth upstream and, to a slight extent, to the increasing thermal conductivity of the water with the decreasing salinity in this direction.

*Salinity.* The Patuxent River estuary is upper estuarine, according to Klugh's (1923) classification of estuaries on the basis of a salinity greater or less than 15°/₀₀, as the salinity of the water is generally below this value.

The salinity of the surface water ranges from an average of 13.8°/₀₀ at Station 6 to an average of 0.7°/₀₀ at Station 24 (Table II). Fresh water, encountered at times at the latter station, extends downstream on ebb tides during periods of heavy stream flow. The surface salinity decreases gradually upstream to Station 14, followed by a more rapid decrease to Station 21, and then a gradual decrease until fresh water is encountered. An average surface salinity of 14°/₀₀ occurs near Station 6, 12°/₀₀ near Station 12, 10°/₀₀ near Station 14, 8°/₀₀ near Station 17, 6°/₀₀ near Station 19, 4°/₀₀ near Station 20 and 2°/₀₀ near Station 21. The more rapid decrease of salinity between Station 14 and 21 is due to the influx of fresh water from the several tributaries entering this region of the estuary. That tributaries effect the upstream salinity

TABLE II. Surface and Bottom Salinity (‰) and pH Characteristics

| Station | Depth (m.) | Surface salinity | | | | Bottom salinity | | | | Surface pH | | | Bottom pH | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Min. | Max. | Aver. | Range | Min. | Max. | Aver. | Range | Min. | Max. | Aver. | Min. | Max. | Aver. |
| 1 | 20 | 9.7 | 18.4 | 13.3 | 8.7 | 16.0 | 22.5 | 19.8 | 6.5 | 7.9 | 8.3 | 8.1 | 7.5 | 8.0 | 7.8 |
| 2 | 14 | 8.8 | 17.9 | 13.0 | 9.1 | 11.6 | 22.5 | 16.7 | 8.5 | 7.9 | 8.3 | 8.1 | 7.5 | 8.0 | 7.8 |
| 3 | 8 | 8.6 | 17.4 | 12.9 | 8.8 | 12.1 | 20.1 | 16.0 | 8.0 | 7.9 | 8.3 | 8.1 | 7.6 | 8.0 | 7.8 |
| 5 | 17 | 9.0 | 17.1 | 13.5 | 8.1 | 12.5 | 18.0 | 15.4 | 5.5 | 7.8 | 8.3 | 8.1 | 7.5 | 8.2 | 7.8 |
| 6 | 2 | 5.4 | 20.4 | 13.8 | 15.0 | — | — | — | — | 7.2 | 8.6 | 8.0 | — | — | — |
| 6A | 17 | 8.0 | 18.4 | 12.9 | 10.4 | 9.4 | 19.7 | 14.3 | 10.3 | — | — | — | — | — | — |
| 7 | 24 | 8.1 | 13.4 | 11.3 | 5.3 | 13.0 | 14.7 | 13.8 | 1.7 | 7.9 | 8.1 | 8.0 | 7.6 | 7.8 | 7.7 |
| 8 | 33 | 8.1 | 16.7 | 12.9 | 8.6 | 11.0 | 17.3 | 14.1 | 6.3 | 7.8 | 8.2 | 8.0 | 7.2 | 7.9 | 7.6 |
| 10 | 9 | 7.7 | 16.3 | 12.2 | 8.6 | 11.0 | 16.9 | 13.5 | 5.9 | 7.8 | 8.2 | 8.0 | 7.6 | 7.9 | 7.7 |
| 10A | 6.5 | 9.6 | 16.0 | 12.2 | 6.4 | 11.2 | 16.5 | 13.5 | 4.3 | 7.8 | 8.2 | 8.0 | 7.6 | 7.9 | 7.7 |
| 12 | 15 | 7.5 | 15.5 | 11.5 | 8.0 | 10.7 | 16.5 | 13.3 | 5.8 | 7.7 | 8.0 | 7.9 | 7.5 | 7.7 | 7.6 |
| 13 | 10 | 7.8 | 15.2 | 11.2 | 7.4 | 9.6 | 16.6 | 13.2 | 7.0 | 7.6 | 8.2 | 7.8 | 7.5 | 7.8 | 7.6 |
| 14 | 9 | 6.5 | 14.4 | 9.9 | 7.9 | 9.9 | 15.4 | 12.3 | 5.5 | 7.6 | 8.4 | 7.8 | 7.3 | 8.2 | 7.6 |
| 15 | 11 | 6.3 | 14.2 | 10.4 | 7.9 | 9.3 | 15.0 | 12.4 | 5.7 | 7.5 | 8.1 | 7.7 | 7.4 | 7.9 | 7.5 |
| 16 | 11 | 7.8 | 9.7 | 8.7 | 1.9 | 8.5 | 14.5 | 10.6 | 6.0 | 7.6 | 8.2 | 7.7 | 7.4 | 7.7 | 7.6 |
| 17 | 4.5 | 4.1 | 12.9 | 8.7 | 8.8 | 5.2 | 13.1 | 9.5 | 7.9 | 7.3 | 8.0 | 7.7 | 7.2 | 8.0 | 7.6 |
| 18 | 6 | 6.2 | 9.7 | 7.7 | 3.5 | 7.7 | 11.2 | 9.0 | 3.5 | 7.6 | 8.0 | 7.7 | 7.6 | 7.6 | 7.6 |
| 19 | 3 | 1.0 | 9.9 | 6.4 | 8.9 | 1.0 | 11.3 | 7.0 | 10.3 | 7.2 | 8.0 | 7.6 | 7.2 | 7.8 | 7.5 |
| 20 | 3 | 2.5 | 6.2 | 4.1 | 3.7 | 3.3 | 6.7 | 4.8 | 3.4 | 7.3 | 7.5 | 7.5 | 7.3 | 7.4 | 7.4 |
| 21 | 6 | 0.1 | 7.6 | 2.3 | 7.5 | 0.1 | 8.1 | 2.7 | 8.0 | 7.0 | 7.8 | 7.3 | 6.8 | 7.6 | 7.2 |
| 22 | 9 | 2.2 | 6.5 | 4.5 | 4.3 | 2.3 | 6.8 | 6.7 | 4.5 | 7.1 | 7.3 | 7.2 | 7.1 | 7.3 | 7.2 |
| 23 | 14 | 0.6 | 4.5 | 2.0 | 3.9 | 0.6 | 5.9 | 2.4 | 5.3 | 7.1 | 7.4 | 7.2 | 7.1 | 7.3 | 7.2 |
| 24 | 4.5 | 0.0 | 2.4 | 0.7 | 2.4 | 0.0 | 3.3 | 0.9 | 3.3 | 6.8 | 7.3 | 7.1 | 6.7 | 7.3 | 7.1 |

gradient is indicated by the fact that the surface water at Station 10A (Fig. 41), located inside the mouth of St. Leonard Creek, is characterized by salinities which occasionally are lower than those at Station 12, three miles farther up the estuary. Salinity data at the various stations are listed in Table II.

There is a large variation in salinity at any one place in the estuary, due to variation in stream flow and precipitation and to tidal and convection currents. The maximum and minimum surface salinities at Station 6 during the 98-month period were, respectively, 20.4°/$_{oo}$ and 5.4°/$_{oo}$, a range of 15°/$_{oo}$. The maximum annual range at this station was 13.3°/$_{oo}$. Due to the increasing tidal amplitude upstream, the salinity range increases in this direction to that point in the estuary where the maximum salinity falls below the salinity range occurring farther downstream, and above that point it decreases.

The salinity range of the surface water is larger than that of the bottom water in the lower two-thirds of the estuary and in the region of the bay opposite its mouth, reflecting the greater tidal shift of the surface water. However, in the upper third of the estuary the salinity range of the bottom water is larger, since the lower salinity of the surface water places a greater restriction on its range.

The percentage difference between surface and bottom water salinities in the lower half of the estuary increases upstream, although the water becomes shallower and the current swifter, both of which are conducive to greater turbulence. Percentage differences throughout the paper are determined by dividing the surface-bottom differential of the factor by the value of the factor in the bottom water. The average percentage difference in surface-bottom salinity of the four groups, for trips in which at least one station of each group was visited, are listed in Table I. The reasons for the differences in the groups will be discussed later. It may be mentioned, however, that the salinity of the bottom water depends upon depth as well as upon proximity to the more saline water of the bay. This is indicated by the fact that the salinity of the bottom water at Station 8, 33 meters deep, occasionally equals the salinity of the bottom water at Station 5, only 17 meters deep but four miles nearer the bay.

There is often a transition in temperature and salinity at the mouths of river estuaries flowing directly into the sea that leads to a marked stratification of the water. Since the Patuxent River estuary does not flow directly into the sea, there is no sudden transition in temperature and salinity at its mouth and consequently no marked stratification of the water. The data (Tables I, V) at the three stations in the region of the bay opposite the mouth of the estuary (Fig. 41) show that differences between the estuary and the bay waters in the physical and

chemical factors studied are of no greater magnitude than the differences in these factors between localities of similar distance within the estuary.

The salinity of the surface water at Station 6 begins to decrease on the average in November (Fig. 43, Table V), but the time of the beginning of the decrease may occur in the preceding or following month. The decrease continues on the average until early May, although it may continue until April or June, at which time the surface salinity begins to increase. The lowest surface salinity occurs from March through August, and the highest from September through February. The trends in salinity in the bottom water at Station 6A are similar to those in the surface water at Station 6, being lowest from March to August and highest from September to February (Table V).

The surface salinity at Station 6 and the stream flow of the Patuxent River measured about five miles above Laurel (about 65 miles on a straight line above Station 6) are inversely related, there being, however, a lag of one to two months between seasonal trends in stream flow and salinity (Fig. 43, Table V). Stream flow at the Coast and Geodetic Survey stream-gauge station near Laurel begins to increase in October and to decrease in May. The lag between seasonal changes in stream flow near Laurel and seasonal changes in the salinity of the estuary decreases upstream. In the upper two-thirds of the estuary the lowest salinities occur from January to May, at which time the stream flow at Laurel is greatest. As would be expected, the upper portion of the estuary responds more rapidly to changes in stream flow measured nearer the source of the river than does the lower portion of the estuary.

The salinity of the lower portion of the Patuxent River estuary is effected by the stream flow of other tributaries of the Chesapeake Bay. Beaven (1946) found an inverse relationship between the salinity at Station 6 and the stream flow in the Susquehanna River, the largest tributary of the Chesapeake Bay.

There is a direct relationship between the periods of increasing salinity and south winds and between the periods of decreasing salinity and winds from the northwest (Fig. 43). South winds blow the more saline bay water into the estuary, while northwest winds have the opposite effect.

Seasonal variations of precipitation in the Patuxent River basin, based on the average precipitation at the five Weather Bureau stations located in this area, show little causal relationship with seasonal variations in salinity or stream flow in the Patuxent River (Fig. 43, Table V). The greatest precipitation occurs during the summer, when the stream flow is decreasing and the salinity increasing. The de-

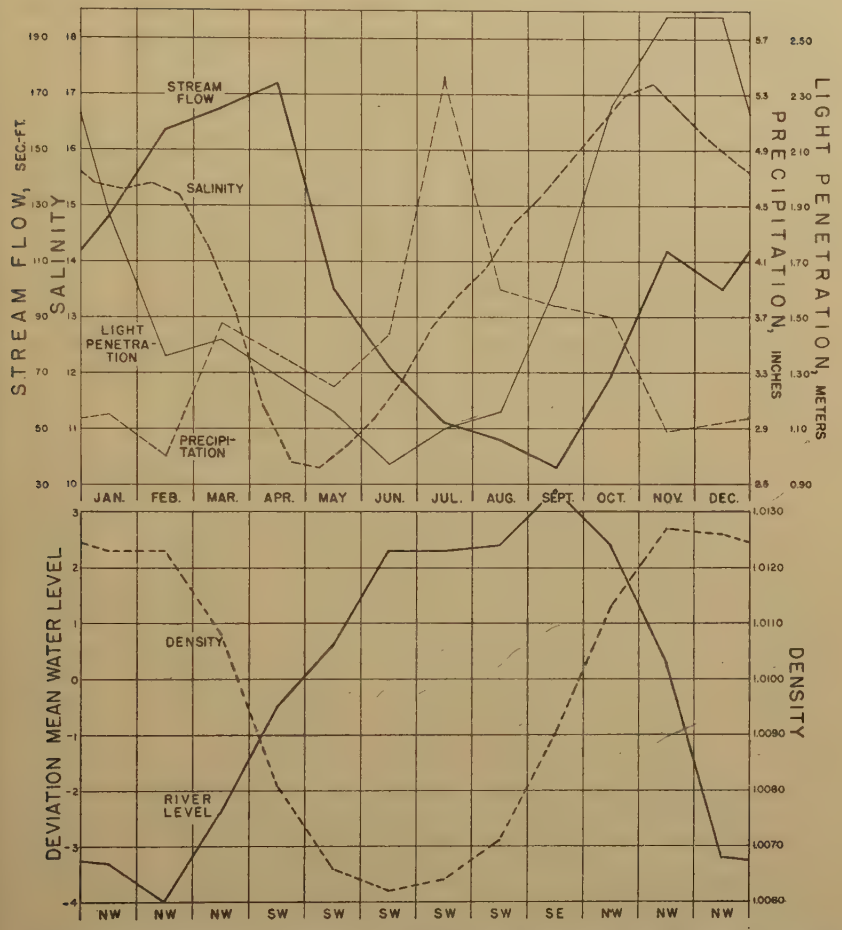


Figure 43. Seasonal variations in the Patuxent River of stream flow, salinity, light penetration, and precipitation in upper diagram, and of density and mean water level deviation (tenths of a foot) in lower diagram.

clining stream flow during the period of greatest precipitation is largely due to the greater evaporation and transpiration occurring during the summer.

The periods of overturn and stratification, indicated by the temperature and density characteristics of the water, are substantiated by the salinity data. The average surface-bottom salinity differential at Station 6A during the period of overturn, September through

February, was 0.9°/₀₀, while during the period of stratification, March through August, it was 1.9°/₀₀. The overturn occurring in the estuary is, because of the salinity of the water, not the complete mixing that occurs in a lake, where, during the period of overturn, the density of the water may become the same throughout. Even at the height of the overturn in the estuary, the bottom water, because of its greater salinity, is usually denser than the surface water. Occasionally, however, the greater warmth of the bottom water causes it to be lighter than the less saline but colder surface water. Also, the tidal effect continually predisposes a mixing of the water which is only relatively diminished during the period of stratification. In Table III are listed the occurrences out of 155 recordings when the surface

TABLE III. Occurrences at Station 6A, 1942–1946, When Surface Salinity Equalled or Exceeded Bottom Salinity

| *Date* | *Surface salinity* (°/₀₀) | *Bottom salinity* (°/₀₀) | *Surface temperature* (°C.) | *Bottom temperature* (°C.) | *Surface density* | *Bottom density* |
|---|---|---|---|---|---|---|
| 9/1/45 | 11.5 | 11.5 | 25.8 | 24.7 | 1.0057 | 1.0061 |
| 10/5/45 | 12.6 | 12.2 | 21.0 | 21.5 | 1.0077 | 1.0073 |
| 11/13/43 | 14.2 | 14.2 | 12.8 | 13.0 | 1.0104 | 1.0104 |
| 11/13/45 | 12.9 | 12.8 | 14.9 | 14.6 | 1.0091 | 1.0090 |
| 1/22/45 | 16.1 | 16.0 | 1.1 | 1.2 | 1.0124 | 1.0123 |
| 1/23/46 | 11.5 | 10.7 | 1.7 | 2.0 | 1.0088 | 1.0082 |
| 2/20/46 | 13.1 | 12.9 | 3.2 | 3.0 | 1.0101 | 1.0100 |
| 2/27/46 | 14.2 | 13.2 | 4.7 | 3.5 | 1.0110 | 1.0102 |
| 4/3/46 | 11.6 | 11.0 | 11.0 | 10.4 | 1.0086 | 1.0082 |
| 4/18/46 | 11.4 | 11.4 | 12.5 | 11.6 | 1.0082 | 1.0083 |
| 5/2/45 | 10.8 | 10.7 | 14.8 | 14.3 | 1.0075 | 1.0075 |
| 8/25/44 | 13.9 | 13.9 | 26.2 | 25.8 | 1.0074 | 1.0075 |

salinity at Station 6A equalled or exceeded the bottom salinity. It can be seen that, while this condition may occur at any season, it is less prevalent during the period of stratification, and on only one occasion during the period of stratification did the surface density exceed the bottom density (Table III).

Salinity, except at Station 6, was calculated from the chlorine content determined by titration with silver nitrate, potassium chromate being used as the indicator. At Station 6, salinity was calculated from the density and temperature of the water with the use of hydrometers calibrated by the Bureau of Standards and with frequent checks by titration.

*Tides.* The difference between the highest and lowest tides at Station 6 during 1945, taken as a representative year, was six feet, about three feet above and three feet below mean river level. The maximum tidal amplitude was 22 inches and the minimum three inches, the average being 13 inches.

*Water level.* The water level at Station 6 is highest from June through September and lowest from December through February (Fig. 43, Table V). The average annual range between the highest mean monthly level, which occurs in September, and the lowest mean monthly level, which occurs in February, is nine inches. The seasonal variations of the water level in the Patuxent River estuary are largely the result of two factors. First, prevailing winds of the region are from the northwest during the period of low water level, and from the south during the period of high water level (Fig. 43). Northwest winds blow the water from the estuary into the bay, while south winds have the opposite effect. Second, the stream flow is high during the period of most rapid increase of water level and low during the period of most rapid decrease of water level (Fig. 43), the river level being determined by the volume of water flowing downstream as well as by the influx of water into the estuary from the bay.

The increase from February to April is largely the result of the high and increasing stream flow occurring during this period, and it occurs in spite of the decreasing salinity, the latter being indicative of an egress of water from the Patuxent River estuary. Outflow of water into the bay is favored by northwest winds in February and March, but it apparently continues into May, as the salinity continues to decrease into this month, despite the seasonal shifting of the wind to the south in April. The south winds begin to retard the egress of the water from the estuary in May, when there is little further drop in salinity, and reverse the direction of the water from the latter half of May through September, during which time the salinity at Station 6 increases. The stream flow begins to decrease after April, and, if it were not for the shifting of the wind to the south, the water level would fall instead of continuing to rise into September.

The river level begins to fall after September, following the seasonal shift in wind direction to the north, despite the increasing stream flow at this time. The effect of wind direction on the egress of water from the Patuxent River estuary is not reflected in the salinity, however, until November, at which time it begins to decrease. Although the stream flow begins to increase during this period, it is not sufficient to cause the water level to rise until after February, and the decrease in river level beginning in the fall is largely due to egress of water from the estuary caused by the northwest winds.

To summarize, the increase in river level from February to April is due largely to high stream flow, and from April to September to the southerly direction of the wind which retards the egress of water from the estuary. The decrease in water level, which occurs from September to February, is due largely to the northwest winds which increase

the rate of egress of water from the estuary. Seasonal variations in direction of wind and amount of stream flow in the Patuxent River estuary are in opposition. During the period of decreasing stream flow the wind is from the northwest, and, during the period of decreasing stream flow the wind is southerly. Thus, what would otherwise be a positive correlation between stream flow and river level is almost obliterated, and the magnitude of seasonal changes in water level is greatly reduced, wind direction having more effect on the river level (measured at Station 6) than has stream flow.

*Dissolved oxygen concentration.* The data of Station 6 and 6A are combined for a study of the seasonal variations in the dissolved oxygen concentration of the surface water. The dissolved oxygen in the surface water begins to decrease in March (Fig. 42, Table V), although high values occur throughout April, and reaches a minimum in August, following the inverse relationship between the solubility of oxygen and the temperature of the water. The maximum concentration recorded was 10.38 c.c./l. and the minimum 2.30 c.c./l.

Similar seasonal variations in the dissolved oxygen concentration occur in the bottom water at Station 6A, the concentration being highest in the winter and lowest in the summer. Lowest values occur during June rather than at the height of water temperature in August. This may be due to the stratification of the water being greater in June than later in the summer, the greatest surface-bottom differentials in temperature, salinity and density also occurring at this time. The maximum oxygen concentration recorded in the bottom water at Station 6A was 9.42 c.c./l. and the minimum 0.15 c.c./l.

Percentage saturation with oxygen of the surface water at Station 6 and 6A is highest in April and May, reaches a low in August, and undergoes a secondary increase in September and October (Fig. 44, Table V). The maximum recorded was 143% and the minimum 42%. Percentage saturation with oxygen of the bottom water at Station 6A undergoes a rapid decrease in May and June, rises somewhat in July and August and sharply in September and October, reaching its highest values in the latter month. The maximum percentage saturation was 133% and the minimum 24%.

The high percentage saturation of the surface water in April and May and the low percentage saturation of the bottom water from May through August are evident in all but the uppermost portion of the estuary. Oxygen saturation values in the bottom water as low as 61% were obtained during the summer as far up the estuary as Station 21. The low percentage saturation of the bottom water as early as May, although the surface water is supersaturated at this time, may

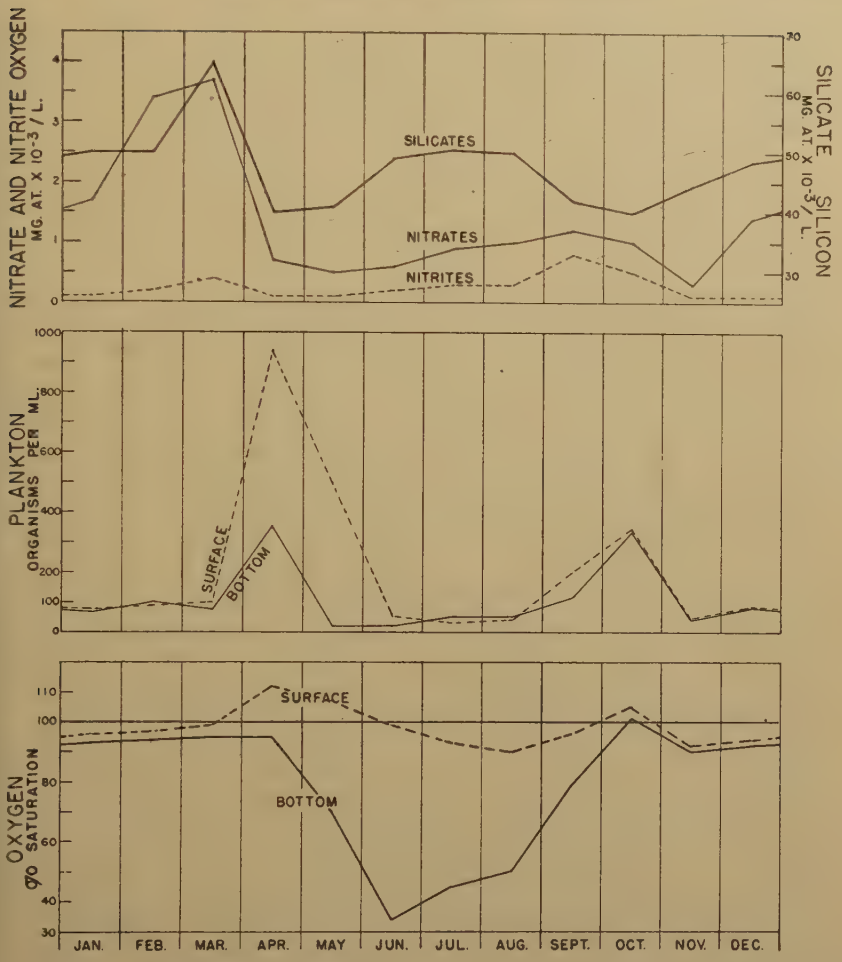


**Figure 44.** Upper diagram—seasonal variations at Group 1 of average surface and bottom concentrations of silicates, nitrates, and nitrites. Middle diagram—seasonal variations at Group 1 of surface and bottom plankton. Lower diagram—seasonal variations at Station 6 and 6A of surface and bottom oxygen percentage saturations.

be due to stratification and bacterial respiration, as well as to the failure of the plankton bloom (predominantly composed of diatoms) to last in the bottom water into this month (Fig. 44).

The surface-bottom differential in dissolved oxygen concentration at Station 6A was 2.2 c.c./l. during the period of stratification and only 0.3 c.c./l. during the period of overturn. The surface-bottom

differential decreases upstream (Table I) due to the increasing shallowness. The greater surface-bottom differential in the bay is due to the greater density stratification, which results in a more severe exhaustion of oxygen in the bottom water during the summer than that which occurs in the estuary.

The dissolved oxygen concentration was determined by a modification of the micro-Winkler method, the reagents being added immediately to the sample upon collection.

*Depth of light penetration.* The Patuxent River estuary would be classified (Minikin, 1920) as a moderately fast flowing river at times of full current velocity, even down to its mouth. Due to tidal action, however, the water is not always moving in the same direction. At change of tide the current velocity is negligible and the rate of sedimentation of suspended particles is greatly accelerated. Characteristically a stream with a current velocity equal to that of the Patuxent River estuary at full strength would have a bed of light gravel, but, due to the tidal effect, its bed is muddy except in the upper reaches where it is sandy.

The depth of light penetration, which was measured by means of a Secchi disc, exhibits a marked increase downstream. The average Secchi disc readings for the four groups, in months in which at least one station of each group was visited, are listed in Table I. The reduction in turbidity downstream is due both to the decreasing current velocity and to the increasing salinity in this direction, the saline water neutralizing and precipitating the negatively-charged suspended particles.

At Station 6, the depth of light penetration is at its lowest in June, 0.97 meter, and at its highest in November and December, 2.58 meters (Fig. 43, Table V). The depth of light penetration, peculiarly, varies inversely with the amount of solar radiation, which is at its height in the latter half of June and is lowest in the latter half of December.

Depth of light penetration is decreased by water with suspended particles flowing downstream, and increased by influx of less turbid bay water into the estuary. Thus, salinity and depth of light penetration are positively correlated seasonally as well as longitudinally. Since salinity varies inversely with the amount of solar radiation, depth of light penetration also exhibits this relationship.

Depth of light penetration shows an inverse relationship with stream flow, there being, however, a lag of two to three months between seasonal changes in stream flow near Laurel and changes in depth of light penetration at Station 6.

*Hydrogen-Ion concentration.* The water of the Patuxent River estuary is characteristically basic, becoming increasingly so downstream (Table II) as the salinity and alkalinity increase in this direction. Alkalinity increases with the greater salinity downstream as well as with vertical increase of salinity, although the alkalinity-chlorinity factor decreases with increasing salinity (Brust and Newcombe, 1940).

The average pH of the surface water at Station 6 is 8.0, with a range of 7.2 to 8.6. A large range in pH occurs throughout the estuary. There is no significant seasonal variation in surface pH at Station 6 (Table V), partly due to the fact that increased alkalinity with the higher salinity in the winter tends to be balanced in its effect on pH by increased solubility of carbon dioxide in the colder water, while decreased alkalinity with the lower salinity in the summer tends to be balanced by decreased solubility of carbon dioxide in the warmer water.

The pH of the bottom water, at any one place in the estuary, is generally lower and never higher than the pH of the surface water, the surface-bottom pH differential decreasing upstream. The surface-bottom pH differential is greater during the period of stratification, when the average differential at Station 5 was 0.4, and lower during the period of overturn, when the average differential was 0.1. The seasonal variation in surface-bottom pH differential is manifested throughout the lower two-thirds of the estuary. The lowest pH of the bottom water occurs during the period of low concentration of dissolved oxygen, June through August, with values as low as 7.5 in the bottom water of the Bay Group.

The higher hydrogen-ion concentration at lower depths may be due in part, as held by Prideaux (1919), to the continued addition of bicarbonate waters to the more saline waters without adequate opportunity of yielding up the excess of carbon dioxide which is produced from the half-bound carbon dioxide by an increase of salinity in solutions already saturated with carbon dioxide. Another source of the lower pH of the bottom water is bacterial respiration. This is particularly evident in the summer when stagnation occurs in the bottom water of the lower half of the estuary and in the region of the bay opposite its mouth. The heat absorbed in the surface water establishes a resistance to mixing, and the reduction in pH is accompanied by a decrease in the dissolved oxygen concentration.

The pH was determined colorimetrically with cresol or phenol-red as indicators, corrections being made for temperature and salinity.

*Silicate concentration.* The range in silicate concentration was $8 \times 10^{-3}$ to $329 \times 10^{-3}$ mg. atoms of Si per liter. The silicate concen-

tration and range of silicate concentration in both surface and bottom waters decrease downstream (Table I). Diatoms fix the silicon in an extremely insoluble form, amorphous opaline silica, so that the concentration of silicates is considerably reduced in the downstream direction. The average silicate concentration in the surface water was 13% higher than the average concentration in the bottom water and exceeded or equalled the latter three-quarters of the time. The surface-bottom silicate differential in Group 1 was higher in the period of stratification, $5.5 \times 10^{-3}$, than during the overturn, $2.0 \times 10^{-3}$.

The major portion of the silicates in the waters of the Patuxent River estuary, because of the low rate of regeneration of silicates from the bottom mud, is derived from the inflowing fresh water. Thus, silicates can be used as a tracer of the fresh water much as salinity is used as a tracer of the bay water carried up the estuary. The greater salinity of the bottom water indicates that the more saline water progressing upstream tends to flow under the fresher water coming downstream. Similarly, the greater amount of silicates in the surface water indicates that the fresher water coming downstream tends to flow over the more saline water progressing upstream from the bay. As is the case with salinity, the greatest percentage difference in the surface-bottom silicate differential occurs in the middle third of the estuary (Table I). Apparently, it is here that separation of inflowing river and bay waters into two strata is at its height.

The higher surface-bottom silicate and salinity percentage differentials in the Bay Group compared with those in Group 1 (Table I) suggest that the greatest relative stratification in the Chesapeake Bay may occur in its middle region (where the Bay Group is located). The salinity of the water in various sections of the bay, determined in July, 1943, indicates this to be true, since the highest surface-bottom salinity percentage differentials occurred in the middle region of the bay, Matapeake to Cove Point (Table IV).

TABLE IV. Salinity in Various Sections of the Chesapeake Bay in July, 1943

| *Location of station* | *Depth (m.)* | *Surface salinity (°/₀₀)* | *Bottom salinity (°/₀₀)* | *Surface-bottom difference (°/₀₀)* | *Surface-bottom difference (°/₀₀)* |
|---|---|---|---|---|---|
| Howell Point | 10 | .50 | .77 | .27 | 54 |
| Hart Island | 5 | 2.82 | 3.05 | .23 | 9 |
| Matapeake | 30 | 7.12 | 18.44 | 11.32 | 160 |
| Sharp's Island | 20 | 8.71 | 19.92 | 11.21 | 129 |
| Cove Point | 35 | 10.74 | 21.75 | 11.01 | 103 |
| Point No Point | 28 | 12.13 | 22.55 | 10.42 | 86 |
| Great Wicomico | 30 | 13.55 | 23.59 | 10.04 | 74 |
| Wolf Trap Light | 15 | 17.17 | 24.38 | 7.21 | 42 |
| Cape Charles | 10 | 23.50 | 31.06 | 7.56 | 32 |

The silicate concentration in Group 1 is highest in March (Fig. 44, Table V) during the heavy stream flow and immediately preceding the spring plankton bloom. It is low in April and May and again in September and October, these periods coinciding with the spring and fall plankton blooms, respectively. The silicate concentration in this group is high during the summer and winter. In Group 3 the silicate concentration more closely follows the stream flow and is lowest during the summer and highest in the spring (Table V).

The colorimetric procedure used by King (1931) was employed for the determination of the silicate concentration.

*Nitrite concentration.* The concentration of nitrite nitrogen ranges from 0.0 to $1.5 \times 10^{-3}$ mg. atoms of N per liter. The average concentration is higher in the lower half of the estuary and in the Bay Group, $0.2 \times 10^{-3}$ in the surface and $0.3 \times 10^{-3}$ in the bottom water, than in the upper half of the estuary, $0.1 \times 10^{-3}$ surface and bottom. In the upper half of the estuary there is no significant difference between surface and bottom water concentrations of nitrites, but in the lower half and in the Bay Group the concentration in the bottom water is 60% higher than in the surface water, and exceeds or equals the concentration in the surface water 85 per cent of the time. The higher nitrite concentration in the bottom water in the lower half of the estuary and in the region of the bay opposite its mouth is indicative of a greater regeneration of nitrites in the more muddy bottom which occurs there.

The nitrite concentration in Group 1 increases gradually from May through September (Fig. 44, Table V), with highest values occurring in the latter month. There is a decrease from September to November, followed by a gradual rise to March and a decrease in April. In Group 3, the nitrite concentration more closely follows the stream flow (Table V).

The surface-bottom differential in Group 1 is greater during the period of stratification, $0.3 \times 10^{-3}$ mg. atoms per liter, than during the rest of the year, when the average surface-bottom differential is $0.0 \times 10^{-3}$.

The nitrite nitrogen concentration was determined by the Griess method as described by Orr (1926–1927).

*Nitrate concentration.* The concentration of nitrate nitrogen is extremely variable, ranging from 0.0 to $8.2 \times 10^{-3}$ mg. atoms of N per liter. The concentration decreases downstream (Table I) due to the greater influx of nitrates in the fresh water and to their gradual utilization in their passage down the estuary. The concentration is 60%

higher in the bottom water and exceeds or equals the surface concentration 80 per cent of the time. The surface-bottom differential is greater in the lower third of the estuary and in the bay (Table I), as nitrate regeneration is favored by the more muddy bottom occurring there.

The nitrate concentration in Group I increases from November to March, reaching its highest values in the latter month (Fig. 44, Table V). There is a sudden drop in April, with lowest values occurring in May and with a gradual increase from May to September. The nitrate concentration generally greatly surpasses the nitrite concentration, and this difference is more pronounced in the winter. Stream flow, which carries a greater proportion of nitrates than nitrites into the estuary, is high during this period, and the low temperature prevailing in the winter has a greater retarding effect upon denitrifying bacteria than upon nitrate-producing bacteria (Rakestraw, 1936). In Group 3 the nitrate concentration more closely follows the stream flow and is highest in the spring (Table V). Concentrations of nutrient salts are dependent upon seasonal changes of stream flow as well as upon regeneration.

The surface-bottom nitrate differential in Group 1 is greater during the period of stratification, $0.9 \times 10^{-3}$, than during the period of overturn, $0.4 \times 10^{-3}$. Because of the tidal currents and relatively shallow depth of the estuary, at least in its upper portion, the existence of seasonal periods of stratification and overturn may have remained unsuspected. Their existence in the lower portion of the Patuxent River estuary is established by the seasonal changes in the surface-bottom differentials of the several factors: density, temperature, salinity, dissolved oxygen, hydrogen-ion concentration and nutrient salts. That the seasonal periods of stratification and overturn also occur upstream, at least through the estuarine portion of the river, is indicated by the seasonal changes in surface-bottom differentials in Group 3. In this group, during the period of stratification, the average surface-bottom differential of density was 0.0006 and of dissolved oxygen 1.1 c.c./l., while during the period of overturn the average surface-bottom differential of density was 0.0003 and of dissolved oxygen 0.7 c.c./l. The seasonal periods of stratification and overturn are indicated even at the uppermost station in the estuary. At Station 24, during the period of stratification, the average surface-bottom differential of density was 0.0003 and of dissolved oxygen 1.3 c.c./l., while during the period of overturn the average surface-bottom differential of density was 0.0000 and of dissolved oxygen 0.5 c.c./l.

Stratification from March through August and overturn from

September through February is also characteristic of the region of the Chesapeake Bay opposite the mouth of the Patuxent River, and is more greatly in evidence there than in the Patuxent River estuary. In the Bay Group, during the period of stratification, the average surface-bottom differential of density was 0.004 and of dissolved oxygen 3.0 c.c./l., while during the period of overturn the average surface-bottom differential of density was 0.001 and of dissolved oxygen 0.4 c.c./l.

Concentrations of phosphates (Newcomb and Brust, 1940), silicates and nitrates are higher in the waters of the Patuxent River estuary than in the waters of the Chesapeake Bay near the mouth of the river (Table I), so that in respect to nutrient salts the bay water has a diluting effect on the water of the estuary, and the estuarine water enriches the bay. Differences in the concentrations of the nutrient salts between the estuary and bay waters, however, are of no greater magnitude than between localities of similar distance within the estuary.

The nitrate nitrogen concentration was determined by the reduced strychnine method outlined by Ridell (1936), the nitrite concentration being deducted.

*Plankton.* The quantity of plankton was determined by the following method. The plankton in a water sample of known volume, about 350 ml., was concentrated by means of a Foerst electric centrifuge, and a 1 ml. portion of the concentrate was placed in a Sedgwick Rafter counting cell. Ten fields were selected at random for counting microplankton, and the whole cell was examined for macroplankton.

While the data are sufficient to show longitudinal differences in the plankton forms, no attempt is made to define seasonal variations in portions of the estuary other than Group 1. A large spring plankton bloom in Group 1 occurs during April and May (Fig. 44, Table V), the highest record of which was 940 organisms per ml. A smaller fall plankton bloom occurs in September and October, the highest record of which was 345 organisms per ml. Quantity of plankton in the bottom water is about the same in both spring and fall blooms, and the supremacy of the bloom in the spring is due to the greater abundance of plankton in the surface water at that time. Lowest values of plankton occur during the summer, particularly in the bottom water where counts as low as 16 organisms per ml. were obtained, plankton being more abundant in the winter than in the summer in both surface and bottom water.

Plankton was more plentiful on the average in the surface than in the bottom water. The difference, however, is largely due to the

abundance of plankton in the surface water during the spring bloom (Fig. 44), there being little difference between the quantities of plankton in the surface and bottom waters during the rest of the year.

While diatoms exceed protozoa in quantity during most of the year throughout the estuary in bottom as well as in surface water, protozoa show an inverse correlation with salinity and increase in quantity upstream. Protozoa are most abundant during the spring and fall blooms with counts as high as 95 per ml., although relatively they are most abundant during the summer when diatom counts are low. Dinoflagellata and Tintinnidae are the predominant protozoan forms. Copepods increase in quantity downstream, while rotifers increase upstream. Copepods are most abundant during the spring bloom when counts as high as 200 per liter were obtained, and rotifers in the fall bloom with counts as high as 750 per liter.

No longitudinal differences in total plankton quantity were observed within the estuary or between the estuary and the region of the bay opposite its mouth. Some of the plankton species occurring in the bay at the mouth of the river did not extend farther up the estuary than Station 5, while others extended upstream as far as Station 19. Some species occurring at Station 24 extended downstream only to Station 21, while others extended down the estuary to Station 17. In general, fresh water plankton fails to reach the lower portion of the estuary and bay plankton fails to reach the upper portion of the estuary. While some species occurred in the middle region of the estuary which were not found in the upper or lower regions, they may have been carried into the middle region by tributaries of the estuary. However, because of the slow rate of progress of the water downstream due to the continually changing tidal direction, it would be possible for a plankton species to exist indefinitely within the middle region of the estuary. Many of the individuals belonging to the species would be carried up and downstream by the tides, but a sufficient quantity could remain within the middle region of the estuary to perpetuate the species without recourse to an external source of supply.

Apparently temperature, other than by producing an overturn, has little direct effect on the initiation of plankton blooms, except that, in the case of both blooms, the water is proceeding towards its average and, what may be, its more favorable temperature. The temperature of the water at the times of the spring and fall plankton blooms indicates that an increase in the quantity of plankton may occur within a range of 5° to 25° C.

Day length and depth of light penetration both may have an effect on the initiation of the plankton blooms. In the spring bloom the days are becoming longer and the depth of light penetration is de-

creasing, while in the fall bloom the days are shortening and the depth of light penetration is increasing.

Nitrates and silicates reach their maxima immediately before the spring bloom and are decreased during the bloom (Fig. 44). They rise gradually to the time of the fall bloom and are similarly decreased by it. These relationships are also true of nitrites (Fig. 44), except that the fall maximum in nitrites is greater than the maximum in the spring. Stream flow, which brings these nutrient salts into the estuary, is at its height immediately preceding and during the early part of the spring bloom, while it is at its lowest preceding and during the early part of the fall bloom. The overturn occurring during the fall bloom, which brings nutrient salts to the surface water and carries dissolved oxygen to the bottom water, apparently partially compensates for the low stream flow at this time. That the fall plankton bloom is as great in the bottom water as it is in the surface water is a manifestation of the overturn.

As the plankton blooms are predominantly composed of plants, the times of their occurrence and the periods of highest oxygen saturation of the water are closely related (Fig. 44). The amount of plankton in the surface water exceeds the amount of plankton in the bottom water by less in the fall bloom than in the spring bloom, and these relationships are reflected in the oxygen percentage saturation values of the water. That depth of light penetration is much less at the time of the spring bloom than at the time of the fall bloom partially accounts for the greater surface-bottom plankton differential in the earlier bloom. The drop in percentage oxygen saturation following the plankton blooms, which is particularly evident in the bottom water during the summer, is apparently largely due to bacterial respiration, as phytoplankton is comparatively abundant even in the oxygen-poor bottom water of the summer period.

## SUMMARY

Longitudinal gradients, surface-bottom differentials and seasonal variations of environmental factors and plankton in the lower and estuarine half of the Patuxent River and in the region of the Chesapeake Bay opposite its mouth are defined from data obtained during 1936 to 1946 at 23 stations.

A single period of stratification from March to August and a single period of overturn from September to February occur throughout the length of the estuary and in the region of the Chesapeake Bay opposite its mouth. Temperature range and earliness of seasonal temperature change increase upstream, the temperature range being greater in

the surface than in the bottom water. The greatest relative stratification of inflowing river and bay waters occurs in the middle third of the estuary, although, because of increasing depth, surface-bottom differentials increase downstream. Surface water on the average is 0.5° C. warmer than the air.

Salinity range is greater in the surface water. Salinity is lower in the spring and summer and higher in the fall and winter, while river level is lower in the winter and higher in the summer. They are both effected by wind direction and stream flow. The effect of seasonal changes of stream flow on the river level is diminished by opposing seasonal changes in the wind direction.

Depth of light penetration increases downstream and, peculiarly, varies inversely with seasonal changes in solar radiation. It is positively correlated with salinity both seasonally and longitudinally. Hydrogen-ion concentration decreases downstream and is greater in the bottom water, particularly in the summer.

Plankton is extremely abundant throughout the estuary and is comparatively plentiful even in the oxygen-poor bottom water during the summer. In the lower third of the estuary the spring plankton bloom in the surface water, occurring in April and May, is considerably greater than the fall bloom, occurring in September and October, while the blooms are of about the same size in the bottom water. The spring bloom is greater in the surface than in the bottom water, while the fall bloom is of about the same size at both depths. Winter values exceed those of summer. Diatoms are the predominant organisms throughout the estuary. Rotifers and protozoa increase in quantity upstream, while copepods decrease in that direction. Protozoa and copepods reach their maxima in the spring bloom and rotifers in the fall. Generally, fresh water plankton fails to reach the lower portion of the estuary, and bay plankton fails to reach the upper portion of the estuary. Some organisms were found only in the middle region of the estuary. No longitudinal differences in total plankton quantity were observed within the estuary or between the estuary and the region of the bay opposite its mouth.

Excess of nitrates over nitrites is more pronounced in the winter. Silicates are more abundant in the fresher surface water and nitrites and nitrates in the bottom water. Concentrations of the nutrient salts are reduced by the plankton blooms and are dependent upon stream flow as well as upon regeneration. Silicates and nitrates decrease and nitrites increase in the downstream direction. Differences in physical and chemical factors are no greater between estuary and bay than between localities of similar distance within the estuary.

TABLE V. Seasonal Variations in Chemical and Physical Factors and Plankton Abundance, 1936–1946

| Period | Temperature (°C.) | | | Density | | | Salinity (°/₀₀) | | | Oxygen (c.c./l.) | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Air St. 6 | Surf. St. 6 | Bot. St. 6A | Surf. 6 | Surf. 6A | Bot. 6A | Surf. 6 | Surf. 6A | Bot. 6A | Surf. 6–6A | Bot. 6A |
| Jan. 1–15 | 2.5 | 3.4 | 3.4 | 1.0123 | 1.0103 | 1.0111 | 15.4 | 13.3 | 14.3 | 8.30 | 8.20 |
| 16–31 | 2.7 | 2.7 | 2.7 | | | | 15.3 | | | 8.22 | 8.13 |
| Feb. 1–14 | 3.5 | 2.8 | 2.7 | 1.0123 | 1.0115 | 1.0121 | 15.4 | 14.9 | 15.5 | 8.30 | 7.95 |
| 15–28 | 3.9 | 3.4 | 2.8 | | | | 15.2 | | | 8.50 | 8.33 |
| Mar. 1–15 | 5.0 | 4.3 | 4.1 | 1.0108 | 1.0087 | 1.0105 | 14.3 | 11.3 | 13.6 | 8.20 | 8.14 |
| 16–31 | 9.5 | 7.3 | 5.5 | | | | 13.1 | | | 7.90 | 7.52 |
| Apr. 1–15 | 10.9 | 9.9 | 9.0 | 1.0081 | 1.0080 | 1.0092 | 11.4 | 11.0 | 12.3 | 8.15 | 7.27 |
| 16–30 | 14.1 | 13.1 | 11.8 | | | | 10.4 | | | 8.07 | 6.60 |
| May 1–15 | 17.6 | 16.5 | 14.9 | 1.0066 | 1.0074 | 1.0086 | 10.3 | 11.4 | 12.5 | 7.09 | 5.06 |
| 16–31 | 21.0 | 20.2 | 18.0 | | | | 10.7 | | | 6.53 | 4.46 |
| June 1–15 | 23.6 | 23.1 | 19.1 | 1.0062 | 1.0052 | 1.0081 | 11.2 | 9.9 | 12.8 | 6.02 | 2.60 |
| 16–30 | 24.0 | 23.9 | 21.3 | | | | 11.9 | | | 5.54 | 1.34 |
| July 1–15 | 25.8 | 26.2 | 23.6 | 1.0064 | 1.0071 | 1.0078 | 12.8 | 13.3 | 14.0 | 5.17 | 2.06 |
| 16–31 | 27.2 | 27.5 | 26.0 | | | | 13.4 | | | 4.96 | 3.00 |
| Aug. 1–15 | 27.2 | 28.2 | 26.9 | 1.0071 | 1.0058 | 1.0071 | 13.9 | 11.9 | 13.4 | 4.76 | 2.86 |
| 16–31 | 24.9 | 26.7 | 26.5 | | | | 14.7 | | | 4.79 | 2.43 |
| Sept. 1–15 | 23.7 | 25.5 | 26.0 | 1.0090 | 1.0078 | 1.0085 | 15.2 | 13.6 | 14.6 | 5.37 | 4.03 |
| 16–30 | 20.9 | 22.8 | 22.8 | | | | 15.8 | | | 5.46 | 4.35 |
| Oct. 1–15 | 17.4 | 19.9 | 20.0 | 1.0113 | 1.0099 | 1.0103 | 16.4 | 14.7 | 15.3 | 6.03 | 5.65 |
| 16–31 | 14.6 | 16.8 | 17.4 | | | | 17.0 | | | 6.90 | 6.72 |
| Nov. 1–15 | 11.9 | 13.8 | 13.8 | 1.0127 | 1.0111 | 1.0115 | 17.2 | 15.0 | 15.6 | 6.20 | 5.91 |
| 16–30 | 8.6 | 9.7 | 11.0 | | | | 16.7 | | | 6.64 | 6.38 |
| Dec. 1–15 | 5.2 | 7.3 | 7.3 | 1.0126 | 1.0116 | 1.0125 | 16.2 | 15.1 | 16.1 | 7.05 | 6.98 |
| 16–31 | 3.0 | 4.7 | 4.7 | | | | 15.8 | | | 7.89 | 7.55 |
| Average | 14.6 | 15.1 | 14.2 | 1.0096 | 1.0087 | 1.0098 | 13.8 | 13.0 | 14.2 | 6.75 | 5.57 |

TABLE V. Seasonal Variations in Chemical and Physical Factors and Plankton Abundance, 1936–1946—*Continued*

| Period | Oxygen (% sat.) | | River Level (ft.) 6 | Light penetration (m.) 6 | pH 6 | Stream flow (sec.-ft.) | Precipitation (in.) | Silicates (mg. at. × /10⁻³/l.) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Surf. 6-6A | Bot. 6A | | | | | | Surf. Bay | Bot. Bay | Surf. Gr. 1 | Bot. Gr. 1 | Surf. Gr. 2 | Bot. Gr. 2 | Surf. Gr. 3 | Bot. Gr. 3 |
| Jan. 1–15 | 96 | 93 | −.33 | 1.87 | 8.0 | 126 | 3.00 | 10 | 9 | 50 | 50 | 83 | 59 | 137 | 64 |
| 16–31 | | | | | | | | | | | | | | | |
| Feb. 1–14 | 97 | 94 | −.40 | 1.36 | 8.0 | 157 | 2.71 | 12 | 11 | 51 | 49 | 77 | 71 | 75 | 54 |
| 15–28 | | | | | | | | | | | | | | | |
| Mar. 1–15 | 99 | 95 | −.24 | 1.42 | 8.0 | 165 | 3.67 | 26 | 21 | 71 | 59 | 41 | 84 | 148 | 146 |
| 16–31 | | | | | | | | | | | | | | | |
| Apr. 1–15 | 112 | 95 | −.05 | 1.29 | 8.0 | 174 | 3.44 | 22 | 18 | 44 | 36 | 73 | 48 | 132 | 132 |
| 16–30 | | | | | | | | | | | | | | | |
| May 1–15 | 107 | 70 | +.06 | 1.16 | 8.0 | 100 | 3.20 | 18 | 15 | 41 | 41 | 98 | 75 | 149 | 177 |
| 16–31 | | | | | | | | | | | | | | | |
| June 1–15 | 99 | 34 | +.23 | 0.97 | 8.0 | 72 | 3.58 | 19 | 17 | 50 | 48 | 100 | 93 | 84 | 87 |
| 16–30 | | | | | | | | | | | | | | | |
| July 1–15 | 93 | 45 | +.23 | 1.10 | 8.0 | 52 | 5.44 | 21 | 18 | 54 | 47 | 99 | 89 | 90 | 104 |
| 16–31 | | | | | | | | | | | | | | | |
| Aug. 1–15 | 90 | 50 | +.24 | 1.16 | 7.9 | 46 | 3.90 | 19 | 17 | 52 | 48 | 105 | 91 | 89 | 100 |
| 16–31 | | | | | | | | | | | | | | | |
| Sept. 1–15 | 96 | 79 | +.34 | 1.61 | 8.1 | 38 | 3.78 | 16 | 13 | 43 | 41 | 79 | 70 | 87 | 94 |
| 16–30 | | | | | | | | | | | | | | | |
| Oct. 1–15 | 105 | 101 | +.24 | 2.26 | 7.9 | 69 | 3.71 | 15 | 11 | 43 | 37 | 83 | 46 | 92 | 91 |
| 16–31 | | | | | | | | | | | | | | | |
| Nov. 1–15 | 92 | 90 | +.03 | 2.58 | 7.9 | 114 | 2.88 | 16 | 13 | 45 | 44 | 79 | 71 | 127 | 144 |
| 16–30 | | | | | | | | | | | | | | | |
| Dec. 1–15 | 94 | 92 | −.32 | 2.58 | 8.1 | 100 | 2.95 | 14 | 12 | 49 | 48 | 93 | 73 | 138 | 136 |
| 16–31 | | | | | | | | | | | | | | | |
| Average | 98 | 78 | ±.00 | 1.61 | 8.0 | 104 | 3.51 | 17 | 14 | 49 | 46 | 84 | 73 | 112 | 110 |

TABLE V. Seasonal Variations in Chemical and Physical Factors and Plankton Abundance, 1936–1946—*Continued*

| Nitrites, $NO_2$ (mg. at. $\times 10^{-3}$/l.) | | | | | | | | Nitrates, $NO_3$ (mg. at. $\times 10^{-3}$/l.) | | | | | | | | Plankton (org./ml.) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Surf. Bay | Bot. Bay | Surf. Gr. 1 | Bot. Gr. 1 | Surf. Gr. 2 | Bot. Gr. 2 | Surf. Gr. 3 | Bot. Gr. 3 | Surf. Bay | Bot. Bay | Surf. Gr. 1 | Bot. Gr. 1 | Surf. Gr. 2 | Bot. Gr. 2 | Surf. Gr. 3 | Bot. Gr. 3 | Surf. Gr. 1 | Bot. Gr. 1 |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.6 | 1.1 | 1.1 | 1.5 | 2.8 | 2.2 | 2.3 | 2.4 | 40 | 50 |
| 0.2 | 0.2 | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 | 3.0 | 3.5 | 3.4 | 3.4 | 3.1 | 3.0 | 2.8 | 2.9 | 70 | 80 |
| 0.3 | 0.5 | 0.3 | 0.5 | 0.3 | 0.4 | 0.4 | 0.4 | 2.3 | 4.2 | 3.6 | 4.0 | 3.5 | 4.1 | 2.7 | 2.7 | 90 | 45 |
| 0.3 | 0.2 | 0.1 | 0.2 | 0.2 | 0.1 | 0.3 | 0.1 | 0.6 | 0.7 | 0.4 | 1.5 | 2.0 | 1.1 | 3.1 | 3.2 | 940 | 355 |
| 0.1 | 0.2 | 0.1 | 0.2 | 0.0 | 0.1 | 0.0 | 0.0 | 0.2 | 0.5 | 0.2 | 1.3 | 1.1 | 1.5 | 1.9 | 2.0 | 495 | 25 |
| 0.1 | 0.4 | 0.1 | 0.4 | 0.0 | 0.2 | 0.0 | 0.0 | 0.5 | 0.7 | 0.2 | 1.4 | 1.0 | 1.5 | 1.5 | 1.7 | 50 | 20 |
| 0.1 | 0.6 | 0.1 | 0.6 | 0.0 | 0.3 | 0.0 | 0.1 | 0.1 | 1.1 | 0.5 | 1.6 | 0.5 | 1.3 | 0.9 | 0.6 | 30 | 50 |
| 0.1 | 0.6 | 0.1 | 0.6 | 0.0 | 0.3 | 0.0 | 0.1 | 0.7 | 1.2 | 0.7 | 1.3 | 0.7 | 1.6 | 1.0 | 1.7 | 40 | 50 |
| 0.6 | 0.8 | 0.7 | 0.9 | 0.4 | 0.5 | 0.1 | 0.1 | 0.6 | 2.0 | 0.8 | 1.6 | 0.9 | 1.4 | 0.9 | 1.2 | 195 | 120 |
| 0.6 | 0.4 | 0.6 | 0.4 | 0.3 | 0.2 | 0.0 | 0.0 | 0.8 | 1.1 | 0.7 | 1.1 | 0.9 | 1.4 | 1.1 | 1.4 | 345 | 335 |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.6 | 0.5 | 0.1 | 0.6 | 1.3 | 1.4 | 1.9 | 2.1 | 25 | 20 |
| 0.2 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.2 | 0.7 | 1.1 | 1.8 | 2.3 | 2.8 | 2.1 | 50 | 45 |
| 0.2 | 0.3 | 0.2 | 0.3 | 0.1 | 0.2 | 0.1 | 0.1 | 0.8 | 1.4 | 1.0 | 1.7 | 1.7 | 1.9 | 1.9 | 2.0 | 200 | 100 |

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# MOMENTUM AND ENERGY INTEGRALS FOR GRAVITY WAVES OF FINITE HEIGHT

By

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1. *Introduction.* The analytical study of gravity waves of finite height which are periodic and move without change in shape in an ideal irrotational fluid is a subject which has occupied the attention of physicists and mathematicians ever since Stokes (*9*) published his classic paper entitled "On the Theory of Oscillatory Waves" in 1847. However, it was not until 1925 that an existence proof for such waves in the case of a basin of infinite depth was published by Levi-Civita (*4*). The corresponding proof for the case of limited depth was presented by Struik (*10*) in 1926.

Much of the effort that has been expended on this wave problem was directed toward the computation of the exact wave form which results from the physical circumstances involved. In this paper we shall not pursue such a course, but rather we shall seek certain simple integral properties of the wave motion, in a sense, directly from the differential equations governing the dynamic system considered, using only a few general properties of the solutions of these equations. In this process we shall assume, of course, the existence of a detailed solution, relying for this upon the proofs given by the investigators alluded to earlier. In the last section the solitary wave will be studied from the same point of view.

2. *General characteristics of the wave motion.* The waves whose study is contemplated here are plane waves, with the crests straight and with every vertical section normal to the crests presenting the same appearance, namely that given in Fig. 45 by the full lines. The basin is assumed to be of a depth $D$, with a horizontal plane bottom and a horizontal plane free surface when there are no waves present. A constant and uniform vertical force of gravity is assumed to act downward on all the fluid particles. The basin is assumed to be of very large dimensions in the direction normal to the crests, but may be limited by plane parallel vertical walls normal to the crests. All the motions are assumed to take place relative to a nonrotating coordinate system. By-and-large these are the same conditions as were

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assumed by Levi-Civita and Struik, the case of infinite depth being approached by making $D$ very large.

We shall make use of a cartesian coordinate system, with the origin below a crest at the mean level of the fluid, the vertical coordinate $z$ being counted positive upward and the horizontal coordinate $x$ counted positive to the right in Fig. 45.

When waves are present the free surface is deformed more or less as shown, so that at a given instant its shape is given by $z = z_o\,(x)$ where $z_o\,(x)$ is a periodic function which is symmetric about the crests and the troughs. We shall assume that the waves are propagated to the right in the figure. Since this propagation is not accompanied by

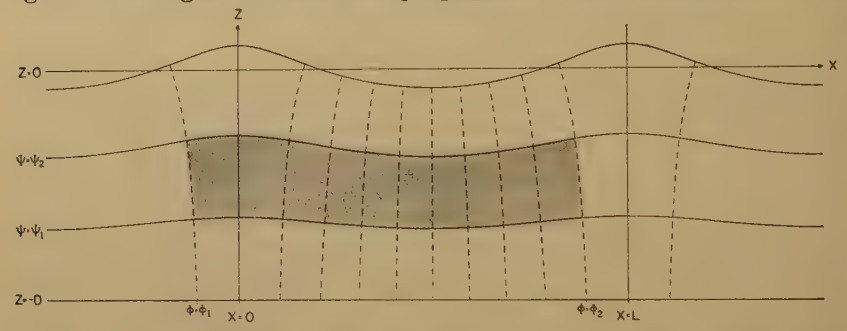


Figure 45. Schematic vertical cross section representing periodic gravity waves in a basin of depth $D$. The full lines are streamlines while the dashed lines are equipotential lines, both for the motion relative to the moving wave. The origin for the vertical coordinate is taken at the mean level of the free surface. The shaded area represents a characteristic region over which various integrals are taken as explained in the text.

any change of structure at the surface or below, it follows that the only effect is to cause a translation of the picture with the (constant) velocity of propagation $c$.

Suppose now that we add a constant horizontal translation, $-c$, to all the particles of fluid involved. The effect of this fictitious added translation to the left makes the wave structure stationary, and the problem is reduced to one of steady motion in a plane. The constant translation does not introduce any change in the internal dynamics of the system and can be allowed for by purely kinematic considerations later.

From the considerations presented above it is apparent that the wave problem can be reduced to a special case of a two-dimensional steady-state jet problem under gravity, having a "free" streamline along which the pressure is constant (or zero) at the top, and a rigid horizontal boundary below. Assuming that, by suitable means, a

proper solution to this jet problem is specified, there still remains a certain arbitrariness in regard to the corresponding progressive wave (except for the case of infinite depth). This difficulty arises from the fact that any constant horizontal translation added to the solution of the jet problem will result in a possible moving wave. This arbitrariness has been mentioned by Stokes (*9*) and also by Lamb (*2*). One convention which can be followed is to define the wave speed so that the resulting progressive waves do not produce a net transport of fluid. We shall, however, adopt an alternative which will be discussed presently.

3. *A momentum integral for periodic gravity waves.* In an article published previously, the writer (*8*) presented a simple derivation of a momentum integral for gravity waves in deep water. An integral of the same nature was first derived by Levi-Civita (*3*) using a different method. A generalization of the writer's approach will now be given for the case of a basin of arbitrary depth.

Considering the steady-state two-dimensional motion represented in Fig. 45, we have at our disposal the continuity equation which may be written in the form

$$\frac{\partial U}{\partial x} + \frac{\partial w}{\partial z} = 0\,, \tag{1}$$

since the fluid is incompressible. In (1) $U$ is the $x$-component of the particle velocity and $w$ is the $z$-component. Since the motion is irrotational, we also have the relationship

$$\frac{\partial w}{\partial x} - \frac{\partial U}{\partial z} = 0\,. \tag{2}$$

The steady-state velocity component $U$ is related to the wave speed $c$, which will be discussed later, by the equation,

$$U = u - c\,, \tag{3}$$

where $u$ is the $x$-component of the particle velocity in the actual progressive wave. The vertical velocity component $w$ is, of course, the same both in the steady motion and in the progressive wave.

By virtue of equations (1) and (2) we may introduce a velocity potential $\Phi$ and a stream function $\Psi$ for the steady motion, such that

$$U = -\frac{\partial \Phi}{\partial x} = -\frac{\partial \Psi}{\partial z}\,; \quad w = -\frac{\partial \Phi}{\partial z} = +\frac{\partial \Psi}{\partial x}\,. \tag{4}$$

Lines along which $\Psi$ is constant constitute streamlines for the steady motion, and are shown by solid lines in Fig. 45. The free surface and

the bottom at $z = -D$ are two members of this family of curves. The lines along which $\Phi$ is constant form a family of orthogonal trajectories to the steady-state streamlines and are shown by dashed lines in Fig. 45. Both $\Phi$ and $\Psi$ are undefined by (4) to the extent of an additive constant in each case.

The velocity components in the progressive wave being $u$ and $w$, it follows from (1), (2) and (3) that we may also introduce a velocity potential $\phi$ and a stream function $\psi$ for the progressive wave, such that

$$(5) \qquad u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial z}; \quad w = -\frac{\partial \phi}{\partial z} = +\frac{\partial \psi}{\partial x}.$$

From (3), (4) and (5) it follows that

$$(6) \qquad \Phi = \phi + cx + k_\phi; \quad \Psi = \psi + cz + k_\psi,$$

$k_\phi$ and $k_\psi$ being constants. Owing to the fact that the actual wave motion is strictly periodic in $x$, we have that, at a given instant,

$$(7) \qquad [U, u, w]_{x+L,z} = [U, u, w]_{x,z},$$

where $L$ is the wave length. Defining the quantities $\Delta\Phi$ and $\Delta\phi$ by the equations

$$(8) \qquad \Phi_{x+L,z} = \Phi_{x,z} + \Delta\Phi; \quad \phi_{x+L,z} = \phi_{x,z} + \Delta\phi,$$

it is seen from (4), (5) and (7) that $\Delta\Phi$ and $\Delta\phi$ are *independent of $x$ and $z$*. As a further consequence of (4), (5) and (7) we may write that

$$(9) \qquad \Psi_{x+L,z} = \Psi_{x,z}; \quad \psi_{x+L,z} = \psi_{x,z}.$$

No constants appear in (9), since the stream function is a measure of transport which must be the same for successive wave lengths by virtue of (7). It follows from (8) and (9) that the pattern of streamlines and equipotential lines for regions separated by an integral number of wave lengths is congruent. From the first equation in (6) and from (8) we obtain the relation that

$$(10) \qquad \Delta\phi = \Delta\Phi - cL.$$

Let us now consider a region in Fig. 45 bounded below by the steady-state streamline $\Psi = \Psi_1$ and above by the streamline $\Psi = \Psi_2$, where the choice of $\Psi_1$ and $\Psi_2$ is arbitrary. Let us further consider a closed portion of this region bounded at the left by the arbitrary equipotential line $\Phi_1$ and at the right by the equipotential line $\Phi_2$ separated from $\Phi_1$ by one wave length so that $\Phi_2 - \Phi_1 = \Delta\Phi$. The kinetic energy $E$ of the steady-state motion represented by this closed region, per unit distance along the crests of the waves, is given by

$$(11)\qquad \frac{2}{\rho}E = \int\int (U^2 + w^2)\,dxdz = \int\int \frac{\partial(\Phi, \Psi)}{\partial(x, z)}\,dxdz = \int\int d\Phi d\Psi = \Delta\Phi\Delta\Psi,$$

where use has been made of (4) and $\rho$ is the (uniform) density.[1a] With the conventions that have been introduced, both $\Delta\Phi$ and $\Delta\Psi$ are positive. In similar fashion it is possible to obtain an expression for the horizontal momentum $M$ of the steady motion represented by the closed region. We thus have that

$$(12)\qquad \frac{1}{\rho}M = \int\int U dxdz = -\int\int \frac{\partial\Psi}{\partial z}\,dxdz = -\int\int dxd\Psi = -L\Delta\Psi.$$

In order to pass from the steady-state motion to the actual motion, we may either transform the integrals $E$ and $M$ by means of (3), or we may write directly that[2]

$$(13)\qquad E = e - cm + \tfrac{1}{2}c^2\mu,$$

and

$$(14)\qquad M = m - c\mu.$$

Here we define $e$, $m$ and $\mu$ as follows:

$$(15)\qquad e = \frac{\rho}{2}\int\int (u^2 + w^2)\,dxdz \quad ; \quad m = \rho\int\int u dxdz \quad ; \quad \mu = \rho\int\int dxdz,$$

so that $e$ is the kinetic energy and $m$ the horizontal momentum of the mass $\mu$ (per unit thickness) in the actual motion. Upon eliminating $\mu$ between (13) and (14) and then substituting from (11) and (12) for $E$ and $M$, and finally by using (10), we obtain

[1a] In general, limits of integration have been omitted where they are clear from the text.

[2] Equation (13) is obtained from the mechanical principle that the kinetic energy of a system of particles is the sum of the kinetic energy due to the motions with respect to the common center of mass and the kinetic energy due to the motion of the center of mass. Thus, if in our problem we denote the first quantity by $T$ and the velocity of the center of mass in the actual motion by $\gamma$, we have that

$$e = T + \tfrac{1}{2}\gamma^2\mu \quad ; \quad E = T + \tfrac{1}{2}(\gamma - c)^2\mu,$$

from which (13) follows when $T$ is eliminated.

$$2e - cm = 2E + cM = \rho(\Delta\Phi - cL)\Delta\Psi = \rho\Delta\phi\Delta\Psi . \tag{16}$$

In the case of an infinitely deep basin, the physically significant solution to the wave problem is one in which all the actual motions disappear at great depths. This means that the constant $\Delta\phi$ must be zero in this case. *We shall make the convention that* $\Delta\phi = 0$ *also for the case of a basin of finite depth.* This convention is equivalent to the relationship that

$$c = \frac{\Delta\Phi}{L} , \tag{17}$$

which we assume to be true for all cases whether the depth be infinite or not. We thus have the two relationships

$$2E + cM = 0 , \tag{18}$$

and

$$\boxed{2e - cm = 0} . \tag{19}$$

The last equation states that the kinetic energy per wave length of the deformed layer considered, when multiplied by two, is equal to the horizontal momentum of the layer per wave length multiplied by the speed of propagation.

There is no limitation in the development which would preclude the possibility of applying (19) to the entire depth of a finite basin or to the total kinetic energy and total momentum for the case of infinite depth.[3]

Let us apply (19) to the entire depth of fluid between the free surface $z_0$ and $-D$. Introducing average values $\tau$ and $\gamma$ defined by

$$\tau = \frac{1}{LD}\int\int \tfrac{1}{2}(u^2 + w^2)\, dxdz , \tag{20}$$

and

$$\gamma = \frac{1}{LD}\int\int u dxdz , \tag{21}$$

so that $\tau$ is the mean kinetic energy divided by the density and $\gamma$ is the mean transport velocity, we may write (19) in the form

[3] Although the region considered above is bounded laterally by equipotential lines for the steady motion, this is not essential. Any region, bounded above and below by lines along which $\Psi$ is constant and whose lateral boundaries are congruent in such a way that they could be brought into coincidence by horizontal translation, may serve equally well, provided that the lateral boundaries are separated by an integral number of wave lengths. This follows from the fact that the more general region may be considered as being composed of a large number of elementary regions of the simple type discussed in the text.

$$\frac{\gamma}{c} = \frac{\tau}{\frac{1}{2}c^2}. \tag{22}$$

Equation (22) was first derived by Levi-Civita (*3*) by other methods. From our analysis it is clear that a relationship of the form (22) is true not only for the entire layer but also for *any* layer which has the shape indicated earlier.

It is of interest to examine what further kinematic implications follow from the choice of $c$ expressed by (17). If the first equation in (4) is integrated along the bottom streamline at $z = -D$, we have, with the aid of (17),

$$\int_{x_1}^{x_1+L} U_D \, dx = -\Delta\Phi = -cL. \tag{23}$$

Using the transformation

$$dx = U_D \, dt, \tag{24}$$

where $t$ is time, we may write

$$\int_{t_1}^{\circ} U_D^2 \, dt = -{}_t\overline{U_D^2}\, t_1 = -cL, \tag{25}$$

from (23), where $t_1$ is the time necessary for a particle to move from $x = x_1 + L$ to $x = x_1$ in the steady motion, and ${}_t\overline{U_D^2}$ is the time average of the square of the velocity of the particle during this period $t_1$. We may also write

$$\int_{\circ}^{t_1} U_D \, dt = -L = {}_t\overline{U}_D \, t_1, \tag{26}$$

where ${}_t\overline{U}_D$ is the time average of $U_D$ for a particle during the period $t_1$. From (25) and (26) we may now form the relation

$$c \, {}_t\overline{U}_D = -{}_t\overline{U_D^2}. \tag{27}$$

By averaging (3) over the period $t_1$ we may write (27) in the form

$$c \, {}_t\bar{u}_D = {}_t\overline{u_D^2}, \tag{28}$$

showing that ${}_t\bar{u}_D$, the average particle velocity at the bottom in the actual wave, is non-negative so that the particles progress in the direction of propagation when waves are present.

From (23), when combined with (3), it follows that

$$\int_{x_1}^{x_1+L} u_D \, dx = 0 . \tag{29}$$

Since in irrotational motion all closed circuits must have zero circulation, it follows from (29) that the line integral from $x = 0$ to $x = L$ of the velocity in the progressive wave along *any* curve joining these verticals (which are located at crests) is zero, or that

$$\int_0^L (u dx + w dz) = 0 . \tag{30}$$

4. *An energy integral for periodic gravity waves.* For the purposes of the subsequent discussion it is necessary to consider the dynamical aspects of the motions studied. The dynamical properties of the motions are, in general, specified by the two hydrodynamical equations of motion for the $x$ and $z$-directions and the equation of continuity (1). For a nonrotating coordinate system the two former equations, in the order mentioned, are:

$$\rho \frac{dU}{dt} = -\frac{\partial p}{\partial x} ; \quad \rho \frac{dw}{dt} = -g\rho - \frac{\partial p}{\partial z} . \tag{31}$$

Here $p$ is pressure and $g$ is the acceleration of gravity. If we introduce a potential $\kappa$ defined by

$$\kappa = p + g\rho z , \tag{32}$$

equations (31) become

$$\rho \frac{dU}{dt} = -\frac{\partial \kappa}{\partial x} ; \quad \rho \frac{dw}{dt} = -\frac{\partial \kappa}{\partial z} . \tag{33}$$

Equations (33) possess an integral which we may write as

$$\frac{\rho}{2}(U^2 + w^2) + \kappa = \frac{\rho}{2} c_1^2 , \tag{34}$$

which is a statement of Bernoulli's equation for the steady-state motion in our problem. In general the constant $c_1^2$ may vary from streamline to streamline, but for the case of irrotational motion it is the same for the whole plane.

Let us rewrite (34), first in the form

$$\frac{\rho}{2}(U^2 + w^2) + \frac{\partial \kappa z}{\partial z} - z\frac{\partial \kappa}{\partial z} = \frac{\rho}{2} c_1^2 , \tag{35}$$

and then, with the aid of the second equation in (33), in the form

$$\frac{\rho}{2}(U^2 + w^2) + \frac{\partial \kappa z}{\partial z} + \rho z \frac{dw}{dt} = \frac{\rho}{2} c_1^2 , \tag{36}$$

or

$$\frac{\rho}{2}(U^2 - w^2) + \frac{\partial \kappa z}{\partial z} + \rho \frac{dzw}{dt} = \frac{\rho}{2} c_1^2 . \tag{37}$$

We shall now integrate (37) over the area mentioned in the previous section, bounded by two streamlines and two equipotential lines separated by one wave length, so that

$$E_x - E_z + \int\int \frac{\partial \kappa z}{\partial z} dxdz + \rho \int\int \frac{dzw}{dt} dxdz = \tfrac{1}{2} c_1^2 \mu , \tag{38}$$

where $E_x$ and $E_z$ are the kinetic energies of the horizontal and vertical components of motion respectively, in the steady-state motion over the area taken. From (34) and (7) we know that

$$[\kappa z, wz]_{x+L,z} = [\kappa z, wz]_{x,z} . \tag{39}$$

It can easily be shown that under these circumstances the last integral on the left-hand side of (38) vanishes.[4] With the aid of Stokes theorem and (39) we may write that

$$\int\int \frac{\partial \kappa z}{\partial z} dxdz = -\oint \kappa z dx = \int_L \kappa_2 z_2 dx - \int_L \kappa_1 z_1 dx , \tag{40}$$

where the contributions of the lateral boundaries vanish because of the periodicity of $\kappa z$ over a wave length. We may now eliminate $\kappa$ by means of (32) and substitute the result in (38) so that

[4] Let us consider any property $H$ characteristic of the steady-state picture. We may write that

$$\frac{dH}{dt} = U\frac{\partial H}{\partial x} + w\frac{\partial H}{\partial z} = \frac{\partial}{\partial x}(UH) + \frac{\partial}{\partial z}(wH) ,$$

the last equality being obtained with the aid of (1). If we now perform an integration over the area involved, we have by Stokes theorem that

$$\int\int \left[\frac{\partial}{\partial x}\left(UH\right) + \frac{\partial}{\partial z}\left(wH\right)\right] dxdz = \oint \left(UHdz - wHdx\right).$$

Since the top and bottom of the area are streamlines along which $Udz - wdx = 0$, no contribution results from the integration along these portions of the circuit. If, further, the quantity $H$ is periodic in $x$, as is the case with $U$ and $w$, the contributions from the remaining two portions of the circuit will cancel so that the value of the entire integral is zero. These conditions are fulfilled when $H = wz$.

$$(41) \quad E_x - E_z + \int_L (p_2 z_2 - p_1 z_1)\, dx + g\rho \int_L (z_2^2 - z_1^2)\, dx = \tfrac{1}{2} c_1^2 \mu .$$

By virtue of the fact that the vertical velocity $w$ is the same in both the actual motion and the steady-state motion, we may write that

$$(42) \qquad E_z = e_z ; \quad E_x = e_x - cm + \tfrac{1}{2} c^2 \mu .$$

Here $e_x$ and $e_z$ are the kinetic energies of the horizontal and vertical components of motion in the actual wave over the area considered. The second relation in (42) is obtained by subtracting $e_z$ from both sides of equation (13). Eliminating $E_x$ and $E_z$ from (41), one obtains the result

$$(43) \quad e_x - e_z - cm + \int_L (p_2 z_2 - p_1 z_1)\, dx + g\rho \int_L (z_2^2 - z_1^2)\, dx = \tfrac{1}{2} (c_1^2 - c^2)\, \mu .$$

This equation is true for any material layer of the shape considered, whether the depth is infinite or not. Moreover, it is independent of any convention in regard to the wave speed $c$. In the case of infinite depth, no sensible derangement of the pressure occurs at great distances below the surface, and hence the pressure must be that which is produced hydrostatically when no waves are present. Although it is not necessary to do so, we shall take the pressure at the free surface to be zero for convenience. Accordingly, $\kappa$ in equation (32) must be zero at great depths and $c_1^2$ in equation (34) must be equal to $c^2$, since here $w = 0$ and $U^2 = c^2$. Inasmuch as $c_1^2$ and $c^2$ are constants, it follows that for the case of infinite depth the right-hand side of (43) vanishes for all the material layers and hence also for the case when the entire depth is considered.

The most interesting case arises when (43) is applied to the entire fluid from the free surface $z_o$ to the bottom. The pressure at the top is now zero while $z_1 = -D$, so that

$$(44) \quad e_x - e_z - cm + g\rho \int_L z_o^2 dx + D \int_L \kappa_D dx = \tfrac{1}{2} (c_1^2 - c^2)\, \mu .$$

The term involving $z_o^2$ is twice the potential energy per wave length for the wave motion. It can be shown that the integral of $\kappa$ with respect to $x$ along any steady-state streamline vanishes if the pressure at the free surface is zero.[5] We thus have

[5] Suppose that we integrate the second of the equations (33) over an area bounded by the free surface on top, by a steady-state streamline below and by two equipotential lines separated by one wave length. According to the discussion in the foot-

(45) $$e_x - e_z - cm + 2v = \Delta\,,$$

in which $v$ represents the potential energy per wave length and $\Delta = \frac{1}{2}(c_1^2 - c^2)\,\mu$. Eliminating $cm$ by means of (19) we have

(46) $$\boxed{e_x + 3e_z + \Delta = 2v}\,.$$

In this relationship $\Delta = 0$ for the case of infinite depth and the equation itself becomes *an exact integral for periodic gravity waves relating the quantities $e_x$, $e_z$ and $v$.* It thus follows that in this case *the sum of the kinetic energy of the horizontal motion and three times the kinetic energy of the vertical motion is equal to twice the potential energy of the waves.*

5. *The partition of energy in periodic gravity waves.* Considering first the case of infinite depth for which $\Delta = 0$, equation (46) may be written in the form

(47) $$e_x - e_z = 2\,(e - v) = 2\varepsilon\,,$$

where $\varepsilon$ is defined as the difference between the kinetic and potential energy of the waves. So far as the writer has been able to find there appears to be no simple means for obtaining the magnitude or algebraic sign of $\varepsilon$ from general considerations without making use of the detailed solution to the wave problem. According to Rayleigh (*7*), who carried out a computation of $\varepsilon$ from the detailed solution, the first term in the power series representing this quantity is positive. This first term is furthermore of the fourth order. Rayleigh's work has been extended by Platzman (*5*) so as to include the term of the eighth order. Platzman's results corroborate those of Rayleigh, and it seems that, to the eighth order of approximation, the kinetic energy of the waves exceeds the potential energy by as much as about twelve per

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note on page 183 this integral will vanish since $w$ is periodic in $x$. We then have by Stokes theorem that

$$-\int\int \frac{\partial \kappa}{\partial z}\,dxdz = \oint \kappa dx = 0\,.$$

Because $\kappa$ is also periodic, no net contribution results from the integration along the lateral boundaries. In the expression (32) for $\kappa$ the pressure along the free surface is zero and the mean level of the free surface is also zero so that no contribution is obtained here. It then follows that the integral of $\kappa$ along the lower streamline must vanish.

This result is merely a statement of the fact that on the average the fluid within a material layer between two streamlines of the steady motion is in hydrostatic equilibrium. It is also possible to arrive at this conclusion by applying the momentum theorem of classical hydrodynamics to the motions of the fluid.

cent when the waves approach the extreme form at the surface. For waves of small amplitude this difference becomes exceedingly small.

Equation (47) gives the result that the kinetic energy of horizontal motion exceeds the kinetic energy of the vertical motion by an amount equal to $2\varepsilon$.

In the case when the surface amplitude is infinitely small with respect to the wave length and also when the depth is small compared to $L$, we know from the small-amplitude theory that the horizontal motion is sensibly the same at all depths and that the kinetic energy of the vertical motions is negligible. This case corresponds to the so-called "long waves." If we integrate (34) along the bottom streamline, we obtain

$$\int_{x_1}^{x_1+L} U_D^2 dx = c_1^2 L \,, \tag{48}$$

since the integral of $\kappa$ vanishes and $w$ is zero. With the aid of (3) and (29) we can also write (48) as follows:

$$\int_{x_1}^{x_1+L} u_D^2 dx = (c_1^2 - c^2)\, L \,, \tag{49}$$

which, incidentally, shows that $\Delta$ is a positive quantity. If we now form the kinetic energy integral for long waves over one wave length, say, between crests, we have

$$e \doteq \frac{\rho}{2}\int_{x_1}^{x_1+L}\int_{-D}^{z_0} u^2 dz dx \doteq \frac{\rho}{2}\int_{x_1}^{x_1+L} u_D^2 (z_0 + D)\, dx \doteq \tfrac{1}{2}(c_1^2 - c^2)\,\mu = \Delta \,, \tag{50}$$

approximately, since $z_0$ is a small quantity. It then follows from (46) that $e \doteq v$. This is in agreement with the results of the small-amplitude theory of long waves.

More generally, for the case of small amplitude but arbitrary depth, we may accept the principle of equipartition of energy given by the classic theory, namely that $e \doteq v$. When this relation is combined with (46) so as to eliminate $v$, we obtain that $\Delta \doteq e_x - e_z$.

A discussion of the most general case—that of arbitrary amplitude and depth—would presumably require an analysis of the detailed solution such as was made by Rayleigh and by Platzman for the case of infinite depth.

6. *Applications to the solitary wave.* Some of the procedures used in this paper may be applied also to the phenomenon of the solitary wave studied principally by Boussinesq (*1*) and Rayleigh (*6*). This (plane) wave consists of a single elevation above the undisturbed level of the free surface and is symmetrical about the crest so that it presents an appearance similar to that shown in Fig. 46. In the case of the extreme amplitude the crest forms a sharp wedge of 120°. The writer is not aware of any direct proof of the existence of this wave similar to the demonstrations of Levi-Civita and Struik for periodic waves. We shall assume, however, that the solitary wave represents a rigorous solution of the dynamic equations of motion, and seek certain integral properties of the wave.

We shall again make use of the artifice of steady motion and consider a region bounded by two steady-state streamlines above and

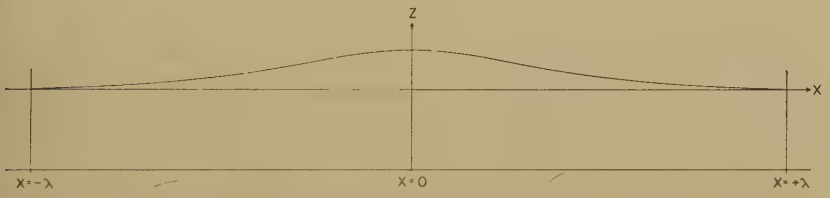


Figure 46 Schematic vertical cross section representing a solitary wave in a basin of depth $D$. The origin for the vertical coordinate is taken at the undisturbed level of the free surface.

below. As the lateral boundaries we shall take two verticals at $x = +\lambda$ and $x = -\lambda$, where $\lambda$ is sufficiently large to place the lateral boundaries in fluid which is not sensibly disturbed. Under these circumstances the vertical boundaries will also be equipotential lines. By means of reasoning analogous to that given in Section 3, we see that the kinetic energy $E$ and momentum $M$ of the steady-state motion are again given by equations (11) and (12), provided we replace $L$ by $2\lambda$. With this slight change equation (16) also holds, but we see that now $\Delta\phi$ cannot be equated to zero, since in the progressive wave all the particles influenced by the wave have a component of motion forward. It therefore follows that equations (18) and (19) are not valid for the solitary wave.

When the entire depth is considered the quantity $\Delta\Psi$ appearing in (12) has, in the present case, a simple interpretation, being equal to $cD$, since the steady motion is a uniform translation for large positive or negative values of $x$. From (12) and (3) we may then write that

$$(51) \qquad -2\lambda cD = \int_{-D}^{z_o} \int_{-\lambda}^{+\lambda} U dx dz = \int_{-D}^{z_o} \int_{-\lambda}^{+\lambda} (u - c)\, dx dz = \frac{1}{\rho} m - 2\lambda cD - c \int_{0}^{z_o} \int_{-\lambda}^{+\lambda} dx dz .$$

If we denote the superficial mass—the mass of the fluid above the undisturbed free surface per unit distance along the crest—by $\mu_o$, it follows that

$$(52) \qquad \boxed{m = c\mu_o} ,$$

where the integrations may be taken over the entire range of $x$. *The momentum of the solitary wave is therefore equal to the momentum of the superficial mass moving with the wave speed.*

In view of the fact that the steady motion is a translation at the rate $c$ at great distances from the crest, the constant $c_1$ in equation (37) must now be equal to $c$, and the other considerations leading to the formulation of this equation are valid for the solitary wave. Furthermore, an equation analogous to (43) is valid, so that when the entire depth is considered (44) may be written as

$$(53) \qquad e_x - e_z - c^2\mu_o + 2v + D \int_{-\lambda}^{+\lambda} \kappa_D dx = 0 ,$$

where use has been made of (52). As explained in the footnote on page 184 we have that

$$(54) \qquad 0 = \oint \kappa dx = \int_{-\lambda}^{+\lambda} \kappa_D dx - g\rho \int_{-\lambda}^{+\lambda} z_o dx ,$$

so that

$$(55) \qquad \int_{-\lambda}^{+\lambda} \kappa_D dx = g\mu_o .$$

Equation (53) thus may be written as

$$(56) \qquad e_x - e_z + 2v + (gD - c^2)\, \mu_o = 0 .$$

The first equation in (31) may be written in the form

$$(57) \qquad \rho \left( \frac{\partial U^2}{\partial x} + \frac{\partial Uw}{\partial z} \right) = - \frac{\partial p}{\partial x} ,$$

because the motion is steady and free of divergence. We shall form the integral of (57) over the area bounded by the bottom, the free surface, the vertical at $x = -\lambda$ and an arbitrary vertical at some value of $x$ within the wave. The area integrals are at once transformable into line integrals by means of Stokes theorem so that we have

$$\rho \oint (U^2 dz - Uw dx) = -\oint p dz. \tag{58}$$

Since no contribution to the integral on the left results from the integration along the top and bottom streamlines, and since the pressure is taken to be zero along the free surface, evaluation of (58) gives that

$$\left[\int_{-D}^{z_0} (\rho U^2 + p)\, dz\right]_x = \left[\int_{-D}^{z_0} (\rho U^2 + p)\, dz\right]_{-\lambda}. \tag{59}$$

The quantity on the right of (59) is immediately capable of simplification, because at $x = -\lambda$

$$U^2 = c^2\,; \quad p = -g\rho z\,; \quad z_0 = 0\,. \tag{60}$$

We thus have that at any arbitrary vertical

$$\int_{-D}^{z_0} (\rho U^2 + p)\, dz = \rho c^2 D + \tfrac{1}{2} g\rho D^2 = \text{constant}. \tag{61}$$

The pressure $p$ may be eliminated from (61) by means of Bernoulli's equation (34), so that, with some reduction,

$$\rho \int_{-D}^{z_0} (U^2 - w^2)\, dz + \rho c^2 z_0 - g\rho z_0^2 = \rho c^2 D\,. \tag{62}$$

If this equation be integrated once more with respect to $x$ from $-\lambda$ to $+\lambda$, it follows that

$$E_x - E_z + \tfrac{1}{2} c^2 \mu_0 - v = \rho c^2 D\lambda\,. \tag{63}$$

We may now pass to the actual motion by using equations (42). Thus

$$e_x - e_z - cm + \tfrac{1}{2} c^2 \mu + \tfrac{1}{2} c^2 \mu_0 - v = \rho c^2 D\lambda\,. \tag{64}$$

Keeping in mind the expression for the momentum $m$ given in (52) and also the fact that $\mu = \mu_0 + 2\rho D\lambda$, we finally have that

$$\boxed{e_x - e_z = v}\,. \tag{65}$$

In the solitary wave *the difference between the kinetic energy of the horizontal and vertical motions is equal to the potential energy.*

By adding $2e_z$ to both members of (65) we have that

$$e = v + 2e_z , \tag{66}$$

so that the kinetic energy is greater than the potential energy.

Equations (56) and (65) may be combined so as to give an expression for the wave speed by eliminating the quantity $e_x - e_z$ between them. The result is

$$\boxed{c^2 = gD + 3\frac{v}{\mu_0}} , \tag{67}$$

showing that the speed of a solitary wave is greater than the speed of the corresponding long waves.

For the highest possible solitary wave it follows from Bernoulli's equation (34) when applied to the free surface that $c^2 = 2gz_T$, where $z_T$ is the height of the crest above the mean level. By inserting this value of $c^2$ in (67), it follows that

$$z_T = \frac{D}{2} + \frac{3}{4}\int z_0^2 dx \Big/ \int z_0 dx . \tag{68}$$

The extreme height is thus greater than half the depth of the basin. The ratio of the integrals in (68) may be interpreted as a mean height $\bar{z}_0$. According to an estimate quoted by Lamb (*2*), $z_T = 0.78D$. The corresponding value of $\bar{z}_0$ turns out to be equal to nearly one-half of $z_T$.

Equations (52) and (65) of this section were derived in such a manner as to be applicable to the entire wave. There is, however, no restriction in the development which would prevent their application to a closed region bounded by *any* two verticals, by the free surface and by the bottom. In the limit we may replace them by integral relationships along a single arbitrary vertical. They then become, respectively,

$$\int_{-D}^{z_0} u dz = cz_0 ; \quad \int_{-D}^{z_0} (u^2 - w^2)\, dz = gz_0^2 . \tag{69}$$

If equations (69) are applied to the vertical at the crest, $w$ vanishes and $z_0$ is the "amplitude" of the wave which we shall denote by $a$. We may then write

$$\bar{u}^2 = c^2 \frac{a^2}{(D+a)^2} ; \quad \overline{u^2} = g\frac{a^2}{D+a} . \tag{70}$$

From the long-wave theory we know that for small surface deformations of large horizontal extent there is no sensible variation of $u$ with depth. For larger amplitudes $u$ varies with depth, so we may write, with the aid of the Schwarz inequality, that $\overline{u^2} \geqq \bar{u}^2$. We then obtain from (70) the result that[6]

$$c^2 \leqq g\,(D + a)\,, \tag{71}$$

where the equality holds for very small values of $a$.

If we retain only the inequality in (71) and eliminate $c^2$ by means of the relation that $c^2 = 2gz_T$ for a wave of extreme height, we get an upper bound for $z_T$, namely $D$. Since we have already found a lower bound we may write that $D/2 < z_T < D$.

## SUMMARY

In this paper certain integral properties of gravity waves in a homogeneous and incompressible fluid are deduced from the differential equations governing such phenomena. The treatment is restricted to plane waves in an irrotational medium which move without change of form. No assumption is made concerning the amplitude so that the results are applicable to waves of finite height. Both periodic and solitary waves are considered. The more important results are the following:

1. For periodic waves the product of the momentum and the wave speed is equal to twice the kinetic energy. This statement is valid not only for the total depth of fluid but also for the fluid contained between any two streamlines of the corresponding steady-state motion.

2. For periodic waves in a basin of infinite depth the sum of the kinetic energy of the horizontal motions plus three times the kinetic energy of the vertical motions is equal to twice the potential energy. Since it has been shown by Rayleigh and by Platzman that the total kinetic energy is greater than the potential energy, this statement implies that the kinetic energy of the horizontal motions is greater than that of the vertical motions.

3. For the solitary wave the momentum is equal to the superficial mass multiplied by the wave speed. This is valid not only for the entire wave but also for any portion of it bounded laterally by two verticals.

4. For the solitary wave the kinetic energy of the horizontal motions is greater than the kinetic energy of the vertical motions by an amount

[6] According to Weinstein (11), the next term in the series representing $c^2$, in terms of $a$, is negative.

equal to the potential energy. From this it follows that the total kinetic energy is larger than the potential energy. These results are valid not only for the entire wave but also for any portion of it bounded laterally by two verticals.

5. For the solitary wave the square of the speed of propagation is equal to the depth of the basin multiplied by the acceleration of gravity plus three times the ratio of the potential energy to the superficial mass.

6. For the solitary wave it can be shown by means of integral relationships that the square of the speed of propagation is less than the height of the crest above the bottom multiplied by the acceleration of gravity, although equality of these two quantities is approached for small amplitudes.

## ACKNOWLEDGMENT

The author wishes to express his gratitude to Mr. G. W. Platzman for many suggestions which have made the presentation of the material contained in this paper clearer and more concise. Also, thanks are due to Messrs. J. Charney and W. E. Baum for many fruitful discussions of the topics treated.

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# THE PARTITION OF ENERGY IN PERIODIC IRROTATIONAL WAVES ON THE SURFACE OF DEEP WATER

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*Introduction.* In the theory of long waves, and also for infinitesimal "surface" waves, the total wave energy is half kinetic and half potential to a first order of approximation.[1] It was pointed out by Rayleigh (1911), in proceeding by successive approximations to the solution of the problem of oscillatory permanent waves on the surface of deep water, that the kinetic energy exceeds the potential by an amount proportional to a quantity of the fourth order, when quantities of order higher than the fourth are neglected.

The exact theory of oscillatory permanent waves was formulated and developed by Stokes (1847), who represented the solution in the form of a Fourier expansion. In this paper are presented general formulas for the kinetic and potential energy in terms of coefficients in the series employed by Stokes, from which the energy may be computed to whatever order these coefficients are known. A review of the general theory is given first, together with the results of a new method for performing the successive approximations that lead to the coefficients in Stokes' solution.

*Review of the General Theory.* The motion, two dimensional and irrotational, is taken in the $xz$ plane, the $x$-axis being horizontal and the $z$-axis vertical, counted positive *upward*. The waves progress in the direction of $x$ *positive*, and the progressive motion is reduced to steady motion by imposing a uniform horizontal velocity in the direction of $x$ negative, of magnitude equal to the wave speed $c$. Thus, if we denote the velocity components of the progressive motion by $u$, $w$, then the corresponding components of the steady motion are $U = u - c$, and $w$. Further, if $\phi$, $\psi$ denote the velocity potential and stream function for the progressive motion, and $\Phi$, $\Psi$ the corresponding functions for the steady motion, then

[1] See Lamb (1932), Art. 174, 230.

$$u = -\frac{\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial z} \qquad w = -\frac{\partial\phi}{\partial z} = +\frac{\partial\psi}{\partial x}$$

$$U = -\frac{\partial\Phi}{\partial x} = -\frac{\partial\Psi}{\partial z} \qquad w = -\frac{\partial\Phi}{\partial z} = +\frac{\partial\Psi}{\partial x}. \tag{1}$$

Hence $\Phi = \phi + cx \qquad \Psi = \psi + cz,$ (2)

provided the origins for the several parameters involved are properly chosen.[2]

We take as the form of the solution that presented by Stokes (1880), in which the variables are inverted, the *independent* variables being $\Phi$, $\Psi$ so that $x$, $z$ are harmonic functions of these. It will prove convenient, in writing this solution and in subsequent calculations, to employ *nondimensional* quantities, defined as follows:

$$\xi = kx \qquad \lambda = k\phi/c \qquad \Lambda = k\Phi/c$$

$$\zeta = kz \qquad \omega = k\psi/c \qquad \Omega = k\Psi/c,$$

where $k = 2\pi/L$, the wave length being $L$. The solution pertaining to the case of deep water is then

$$\xi = \Lambda + \Sigma A_r e^{r\Omega} \sin r\Lambda, \qquad \zeta = \Omega + \Sigma A_r e^{r\Omega} \cos r\Lambda, \tag{3}$$

the summation extending over all *positive* integral values of the index $r$. In the steady motion the free surface is a streamline, which we take to be $\Omega = 0$, while for great depths $\Omega \to -\infty$. Thus, at the free surface[3]

$$\xi_0 = \Lambda + \Sigma A_r \sin r\Lambda, \qquad \zeta_0 = \Sigma A_r \cos r\Lambda, \tag{4}$$

and for great depths $\xi \to \Lambda$ and $\zeta \to \Omega$.

The coefficients $A_r$ in (3) are determined in such a manner as to satisfy the condition of uniform pressure at the free surface, which may be written in the nondimensional form

$$\left(\frac{Q}{c}\right)_0^2 + \frac{2}{\mu}\left(\zeta_0 - \overline{\zeta_0}\right) = 1,$$

where $Q^2 = U^2 + w^2$; and

$$\mu = \frac{k}{g}c^2 = \frac{2\pi c^2}{gL} \tag{5}$$

[2] The symbol $x$ in (2) refers to the horizontal co-ordinate measured relative to the steady motion; equations for the progressive motion may be found at any stage by writing $x - ct$ for $x$.

[3] The subscript $_0$ will be used to identify quantities evaluated at the free surface. A single summation sign $\Sigma$ will stand throughout for $\Sigma_{r=1}^{\infty}$ or $\Sigma_{s=1}^{\infty}$.

is the square of the ratio of the wave speed to the speed of propagation of "infinitesimal" surface waves in deep water ($g$ denoting the acceleration of gravity). The quantity $\overline{\zeta_0}$ is the elevation of the mean level of the free surface,

$$\overline{\zeta_0} = \frac{1}{2\pi}\int_0^{2\pi} \zeta_0 d\xi_0 = \frac{1}{2}\Sigma r A_r^2 . \tag{6}$$

In the method utilized by Stokes, each coefficient $A_r$ is expressed ultimately as a series of powers of the leading coefficient $A_1 = b$. This determination, which proceeds by successive approximations, has been carried to the tenth order by Wilton (1914), whose results (through eighth order) are shown in Table I.[4]

TABLE I. Coefficients $A_{rs}$ (Through Eighth Order) in the Series $A_r = \Sigma A_{rs} b^s$. Where No Entry Is Made the Corresponding $A_{rs}$ Is Null

| $r$ \ $s$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | — | — | — | — | — | — | — |
| 2 | — | 1 | — | 1/2 | — | 29/12 | — | 1123/72 |
| 3 | — | — | 3/2 | — | 19/12 | — | 1183/144 | — |
| 4 | — | — | — | 8/3 | — | 313/72 | — | 103727/4320 |
| 5 | — | — | — | — | 125/24 | — | 16603/1440 | — |
| 6 | — | — | — | — | — | 54/5 | — | 54473/1800 |
| 7 | — | — | — | — | — | — | 16807/720 | — |
| 8 | — | — | — | — | — | — | — | 16384/315 |

From an inspection of Table I it is evident that $A_r = \Sigma A_{rs} b^s$ is of order $r$ at least; in fact, $A_r/b^r$ is a series of even powers of $b$. No general formula for $A_{rs}$ is known, except for $A_{rr}$, which Wilton (1914) has shown to be $A_{rr} = r^{r-2}/(r-1)!$.

The *height* $h$ of the wave (elevation of crest above trough) is given by

$$kh = \zeta_0(\text{crest}) - \zeta_0(\text{trough}) ,$$

where $k = 2\pi/L$. Since $\Lambda = 0$ at a crest and $\pi$ at a trough, we find from (4), for the ratio of height to length,

$$\frac{h}{L} = \frac{1}{\pi}(A_1 + A_3 + A_5 + \ldots) ,$$

[4] The origin for $\Lambda$ is conventionally taken to coincide with a trough, and with this convention the $A_r$-coefficients are found to be alternately positive and negative. However, we here place this origin at a crest, so that the $A_r$-coefficients are *all positive*.

or, writing $\pi h/L = ka$,

$$ka = A_1 + A_3 + A_5 + \ldots, \tag{7}$$

where $a = h/2$ is the wave *amplitude*.

The expansion (3) represents the solution of the wave problem for all waves whose height is less than that of the *highest* wave. Michell (1893) gave an exact theory for the highest wave, which Stokes had shown to have pointed crests, the front and rear slopes of the profile meeting there at an angle of 120 degrees. According to Michell's analysis the ratio of height to length for the highest wave is 0.142 approximately, or nearly 1/7. Havelock (1918), by an elegant extension of Michell's method, obtained an expansion that represents the solution of the oscillatory wave problem for all waves up to and including the highest, thus bringing into harmony the results of Stokes and those of Michell. Further, he found, for the highest wave, the ratio of height to length $h/L = 0.1418$ and the value of the parameter in Stokes' solution $b = 0.2919$.

In the method of approximation employed by Stokes, the dimensionless quantity $\mu$ is obtained as a series of even powers[5] of $b$,

$$\mu = 1 + b^2 + \frac{7}{2}b^4 + \frac{229}{12}b^6 + \frac{6175}{48}b^8 + \ldots$$

The convergence of this series is not rapid for values of $b$ corresponding to waves close to the highest wave. Michell found for the highest wave $\mu = 1.200$.

By defining a quantity $\beta$ according to

$$\mu = 1 + \beta^2$$

it is possible to perform the successive approximations leading to the $A_r$-coefficients in such a manner that each $A_r$ is expressed ultimately as a series of powers of $\beta$. The result of such a determination (which is not more laborious than that of Stokes) is shown in Table II.[6]

[5] The value 7427/48 given by Wilton for the coefficient of $b^8$ is erroneous.

[6] Table II and Table I may be shown to be in agreement by expressing $\beta$ in terms of $b$ and converting the $\beta$-series into $b$-series.

TABLE II. Coefficients $A_{rs}$ (Through Eighth Order) in the Series $A_r = \Sigma A_{rs}\beta^s$. Where No Entry Is Made the Corresponding $A_{rs}$ Is Null

| $r$ \ $s$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | +1 | — | −7/4 | — | +113/96 | — | −949/384 | — |
| 2 | — | +1 | — | −3 | — | +13/3 | — | −1051/144 |
| 3 | — | — | +3/2 | — | −151/24 | — | +7741/576 | — |
| 4 | — | — | — | +8/3 | — | −1031/72 | — | +172457/4320 |
| 5 | — | — | — | — | +125/24 | — | −24511/720 | — |
| 6 | — | — | — | — | — | +54/5 | — | −149647/1800 |
| 7 | — | — | — | — | — | — | +16807/720 | — |
| 8 | — | — | — | — | — | — | — | +16384/315 |

It is to be noted particularly that each of the β-series in Table II is an *alternating* series, and therefore each may be expected to converge more rapidly than the corresponding $b$-series in Table I.

The relation between β and $b$ may be found by equating the expressions for $A_1$ in the two tables,

$$b = \beta - \frac{7}{4}\beta^3 + \frac{113}{96}\beta^5 - \frac{949}{384}\beta^7 + \ldots,$$

or reverting,

$$\beta = b + \frac{7}{4}b^3 + \frac{769}{96}b^5 + \frac{6439}{128}b^7 + \ldots$$

Similarly, by applying (7) for the wave amplitude,

$$ka = \beta - \frac{1}{4}\beta^3 + \frac{3}{32}\beta^5 + \frac{1543}{5760}\beta^7 + \ldots$$

or, writing $\alpha = ka$, and reverting,

$$\beta = \alpha + \frac{1}{4}\alpha^3 + \frac{3}{32}\alpha^5 - \frac{1543}{5760}\alpha^7 \ldots$$

Since $kc^2/g = \mu = 1 + \beta^2$, we find the following series for the wave speed,

$$c^2 = \frac{g}{k}\left(1 + \alpha^2 + \frac{1}{2}\alpha^4 + \frac{1}{4}\alpha^6 - \frac{22}{45}\alpha^8 \ldots\right). \qquad (8)$$

The quantity $\alpha = \pi h/L = ka$ is the product of wave number and wave amplitude (one-half wave height). Since the maximum value of $h/L$, corresponding to the highest wave, is 0.1418, the maximum value of

$\alpha$ is 0.4455. For this value the successive partial sums of the series in (8) are 1.000, 1.198, 1.218, 1.220, 1.219, . . . ., the proper value being 1.200 for the highest wave. Although the convergence of (8) is not very strong for the highest wave, the error in neglecting terms of order higher than the second, that is, the error in the approximate formula

$$c^2 \approx \frac{g}{k}(1 + \alpha^2) = \frac{g}{k}(1 + k^2a^2),$$

is less than two per cent for waves of arbitrary height.

*Kinetic Energy.* The kinetic energy of the *progressive* motion in one wave length is

$$e = \frac{1}{2}\rho \int\int (u^2 + w^2)\, dxdz,$$

the integration extending from $x = 0$ to $L$ (crest to crest) and through the entire depth of fluid ($\rho$ denoting the fluid density, supposed uniform). However, it follows directly from (1) that $u^2 + w^2 = \partial(\phi, \psi)/\partial(x, z)$, and hence

$$e = \frac{1}{2}\rho \int\int d\phi d\psi = \frac{1}{2}\rho \oint \phi d\psi.$$

The line integral may be written in terms of dimensionless quantities,

$$\frac{2k^2}{\rho c^2} e = \oint \lambda d\omega,$$

and evaluated as follows.

Equations (2) in nondimensional form,

$$\Lambda = \lambda + \xi, \qquad \Omega = \omega + \zeta,$$

together with (3), show that

$$\lambda = -\Sigma A_r e^{r\Omega} \sin r\Lambda, \qquad \omega = -\Sigma A_r e^{r\Omega} \cos r\Lambda.$$

It is evident that $\lambda$ becomes vanishingly small at great depths ($\Omega \to -\infty$) and vanishes everywhere along the lateral boundaries of the region of integration, that is, along $\Lambda = 0$ and $\Lambda = 2\pi$. There remains in the line integral only a contribution along the free surface ($\Omega = 0$), where

$$\lambda_0 = -\Sigma A_r \sin r\Lambda, \qquad \omega_0 = -\Sigma A_r \cos r\Lambda,$$

the limits of integration being $\Lambda = 0$ and $\Lambda = 2\pi$.

Proceeding in this way it is easily found that[7]

$$\frac{2k^2}{\rho c^2} e = \pi \Sigma r A_r^2 = 2\pi \overline{\zeta}_o ,$$

where $\zeta_o$ is the elevation of the mean level of the wave profile. Making use of (5) we have finally

$$\frac{2k^3}{\pi \rho g} e = \mu \Sigma r A_r^2 ;$$

expanding the right-hand member as a $\beta$-series,

$$\frac{2k^3}{\pi \rho g} e = \beta^2 - \frac{1}{2}\beta^4 - \frac{4}{3}\beta^6 - \frac{251}{144}\beta^8 . \quad . \quad . \quad . \qquad (9)$$

*Potential Energy.* The potential energy of the wave in one wave length is

$$v = \frac{1}{2}\rho g \int^L (z_o - \overline{z}_o)^2 dx ,$$

or reducing to nondimensional quantities,

$$\frac{2k^3}{\pi \rho g} v = \frac{1}{\pi}\int_0^{2\pi} (\zeta_o - \overline{\zeta}_o)^2 d\xi = 2\overline{\zeta_o^2} - 2\overline{\zeta}_o^2 .$$

In terms of the $A_r$-coefficients $\overline{\zeta}_o$ is given by (6). The evaluation of $\overline{\zeta_o^2}$ is somewhat more laborious but after some reduction is found to give[8]

$$2\overline{\zeta_o^2} = \Sigma A_r^2 + \Sigma\Sigma_{(r=p+q)} r A_p A_q A_r .$$

Thus, for the potential energy,

$$\frac{2k^3}{\pi \rho g} v = \Sigma A_r^2 + \Sigma\Sigma_{(r=p+q)} r A_p A_q A_r - \frac{1}{2}(\Sigma r A_r^2)^2 ;$$

expanding the right-hand member as a $\beta$-series,

$$\frac{2k^3}{\pi \rho g} v = \beta^2 - \beta^4 - \frac{5}{6}\beta^6 - \frac{257}{144}\beta^8 . \quad . \quad . \quad . \qquad (10)$$

*Partition of Energy.* The difference between kinetic and potential energy may be expressed as a $\beta$-series[9] by subtracting (10) from (9):

[7] This result may be obtained directly through an application of the *momentum theorem* developed by Starr (1947).

[8] We make use of the identity $\Sigma\Sigma_{(r=|p-q|)} r A_p A_q A_r \equiv \Sigma\Sigma_{(r=p+q)} r A_p A_q A_r$. The summation extends over all positive integral values of $p$ and $q$.

$$\frac{2k^3}{\pi\rho g}(e - v) = \frac{1}{2}\beta^4 - \frac{1}{2}\beta^6 + \frac{1}{24}\beta^8 - \frac{55}{144}\beta^{10} + \ldots . \quad (11)$$

This result corroborates the computations of Rayleigh (1911), who found that the series for $e - v$ starts with a term of the fourth order.

Dividing (11) by (10) we obtain a series giving the ratio of $e - v$ to $v$,

$$\frac{e - v}{v} = \frac{1}{2}\beta^2 + \frac{11}{24}\beta^6 + \frac{31}{32}\beta^8 + \ldots .$$

This may be developed as an $\alpha$-series by means of the series preceding (8); thus,

$$\frac{e - v}{v} = \frac{1}{2}\alpha^2 + \frac{1}{4}\alpha^4 + \frac{7}{12}\alpha^6 + \frac{2033}{1440}\alpha^8 + \ldots ., \quad (12)$$

where $\alpha$ is the *amplitude parameter ka*. The maximum value of $\alpha$, corresponding to the highest wave, is 0.4455, for which the successive convergents of (12) are, in per cent, 9.93, 10.92, 11.37, 11.59, . . . . Although the convergence is not very rapid, it appears that the maximum value of $(e - v)/v$ is approximately $\frac{1}{8}$.

*Water of Limited Depth.* The foregoing considerations are applicable without alteration when the depth of water is limited, provided only that appropriate coefficients in the Fourier expansions are employed. Stokes (1880) has carried the successive approximations (leading to the $A_r$-coefficients) through the third order in this case, but so far as the writer is aware, these computations have not been extended subsequently. Further, there does not exist in the literature, to the writer's knowledge, an analysis of the highest waves of permanent type when the depth of water is limited, corresponding to the analysis given by Michell for deep water. For these reasons it has not seemed worthwhile to extend the energy computations to the case of limited depth.

## ACKNOWLEDGMENT

The writer owes his interest in Stokes' problem to Professor Victor P. Starr, whose counsel and encouragement have been indispensable. Thanks are extended to Werner A. Baum and Harriet M. Platzman for aid in checking the computations.

[9] A tenth-order term is included in (11) even though such terms are not present in (9) and (10). This is possible without extending Table II, because the tenth-order term in (11) is independent of terms higher than the eighth order in the $A_r$-coefficients.

## SUMMARY

Periodic irrotational waves of permanent type on the surface of deep water are studied for the purpose of analyzing the partition of total wave energy into kinetic and potential. Employing Stokes' representation of the solution of the wave problem, successive coefficients in the Fourier expansion are computed to the eighth order in terms of $\beta = \sqrt{1 - \mu}$, where $\mu = 2\pi c^2/gL$ is the square of the ratio of the wave speed to the speed of propagation of infinitesimal surface waves. An approximate frequency equation is given, which may be written $\mu \approx 1 + (\pi h/L)^2$ (where $h$ is the height of the wave) and which is in error by not more than two per cent for all waves including the highest.

The kinetic energy $e$ and potential energy $v$ are expressed as power series in $\beta$, the determination in each case being carried to the eighth order. A $\beta$-series is then derived for the ratio $(e - v)/v$, from which it is shown that the maximum value of this ratio, corresponding to the highest wave, is approximately $\frac{1}{8}$.


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# A CRITICAL WIND SPEED FOR AIR-SEA BOUNDARY PROCESSES[1]

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## INTRODUCTION

Most processes at the air-sea boundary are modified by the existing wind pattern. In general the rate of these processes varies *gradually* with wind speed. An exception seems to occur at a wind speed of about 7 m/sec; at that speed a number of apparently unrelated processes have been observed to undergo such *abrupt* changes that, in some instances, one is led to suspect the existance of discontinuities. Not only the rate, but the very nature of these processes seems to be altered at this critical wind speed.

An explanation may be found in the Kelvin-Helmholtz instability criterion; applied to the air-sea boundary this criterion gives instability for winds exceeding 6.5 m/sec. The application involves the transition from laminar to turbulent flow, a concept fundamental to fluid mechanics. Consequently oceanographic and meteorologic processes involving the air-sea boundary can be expected to change markedly at the critical wind speed.

## OBSERVATIONS OF "DISCONTINUITIES" AT BEAUFORT 4

*Whitecaps.* Whitecaps[2] are formed when the wind speed equals or exceeds force 4 on the Beaufort scale (Beaufort 4 includes the range 5.5–7.9 m/sec, *i. e.*, 11–16 knots). The transition from a smooth sea to a sea covered by whitecaps is so striking that it serves as one of the principal indications for judging surface winds.

[1] Contribution from the Scripps Institution of Oceanography, New Series, No. 349. This work represents the results of research carried out for the Hydrographic Office, the Office of Naval Research, and the Bureau of Ships of the Navy Department, under contract with the University of California.

[2] "The white froth on the crests of waves" (Knight, 1944: 829). In England the expression "white horses" is commonly used.

Table I contains a description of the surface at various wind speeds. These speeds refer to average values; instantaneous values of the wind

TABLE I. Description of Sea Surface at Various Wind Speeds

| Wind Speed | | Definition* | Description† |
|---|---|---|---|
| Beaufort | Knots | | |
| 3 | 7–10 | Gentle breeze | ". . . produces a definite wave formation with scattered incipient whitecaps making their appearance." |
| 4 | 11–16 | Moderate breeze | ". . . is characterized by numerous well developed whitecaps, which give the ocean a spotted appearance." |
| 5 | 17–21 | Fresh breeze | "The difference in the appearance of the ocean surface between forces 4 and 5 is principally one of degree. However, with force 5 spume tends to be blown from the breaking wave crests, whereas no such tendency is noticeable with force 4." |

* Knight, 1944.

† U. S. Dept. of Agriculture, Weather Bureau. Instructions to marine meteorological observers, 1938.

speed may exceed the average values by as much as 50%. Therefore the lowest instantaneous wind speed sufficient to bring about whitecaps appears to lie in the range 11 to 16 knots.

Plates I, II and III show selected photographs of the sea surface at wind speeds just beneath and just above those required for whitecap formation. The optical properties of the sea surface differ at the two wind speeds. On vertical photographs, such as Plate III, foam patches were counted and their number over an area of approximately 10,000 square meters was plotted against the wind speed (Fig. 47). These counts depend somewhat upon personal judgment in selecting foam patches. There are no foam patches at wind speeds of less than 5 m/sec, but there are from 10 to 15 foam patches[3] for winds exceeding 7 m/sec.

*Soaring of Birds.* Woodcock (1940) has observed the soaring tactics of sea gulls over the open sea. He distinguishes between two types of free soaring: circle soaring, where "birds circle about in horizontally restricted, chimney-like up-drafts, being carried along downwind (if there is a wind) at the same time"; and linear soaring, where "the

[3] Above the critical wind speed the *number* of foam patches does not increase consistently with the wind, but their diameters do increase from 3 feet for a speed of about 7 m/sec, to 25 feet for speeds exceeding 13 m/sec.

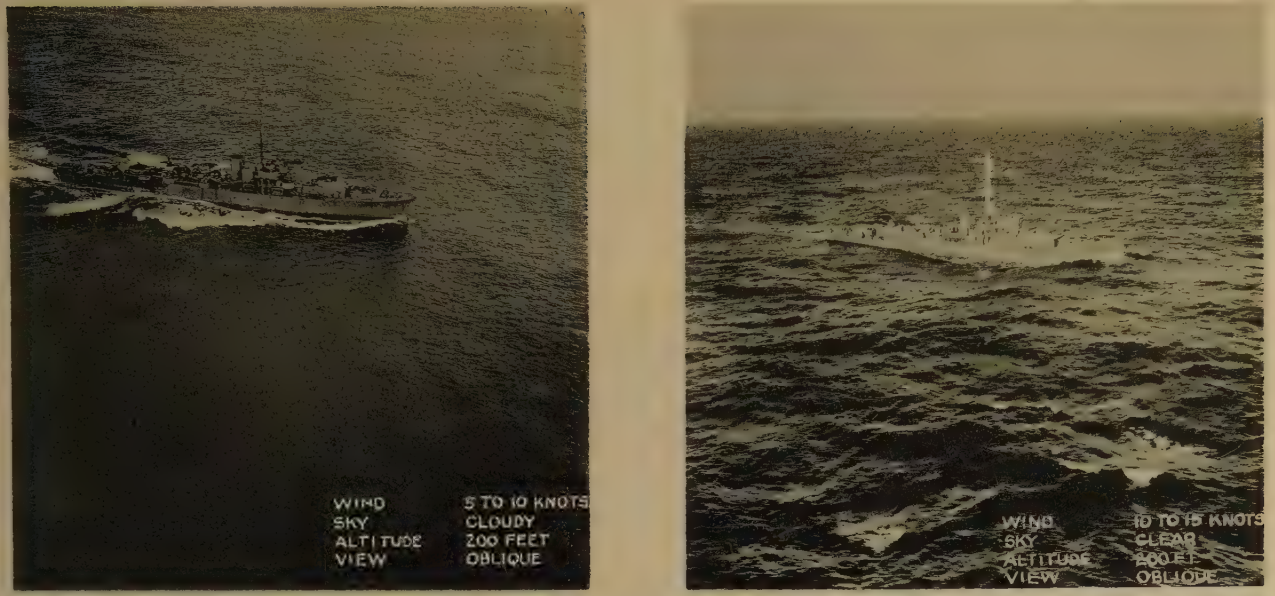


Plate I. Oblique view of the sea surface from an elevation of 200 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)

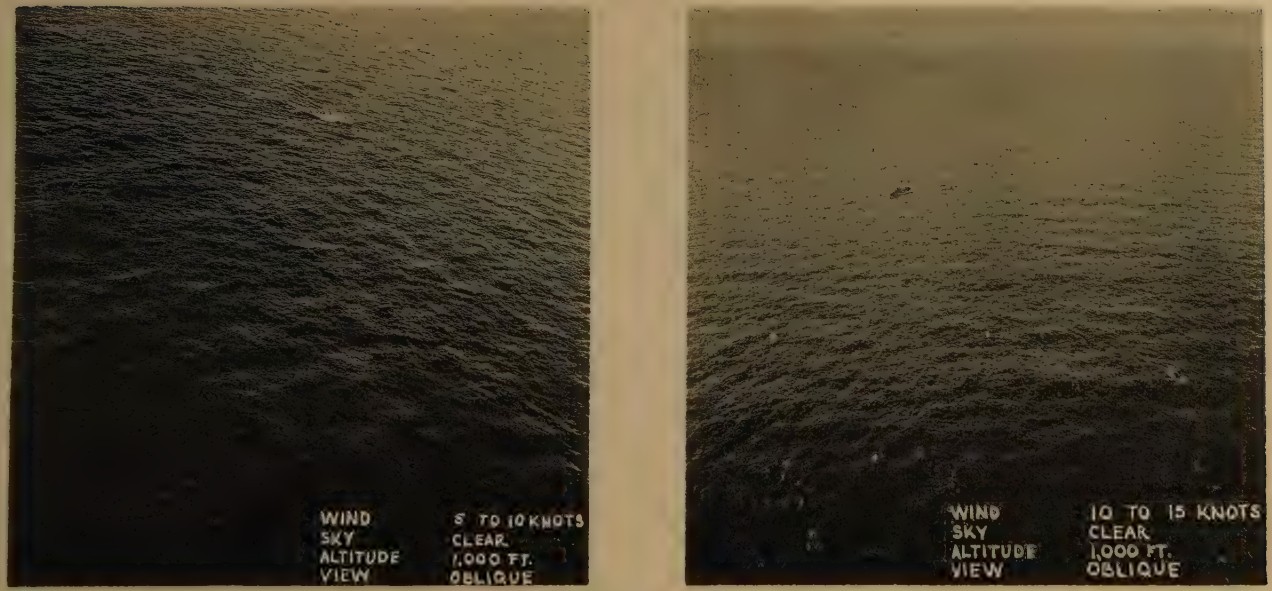


Plate II. Oblique view of the sea surface from an elevation of 1000 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)

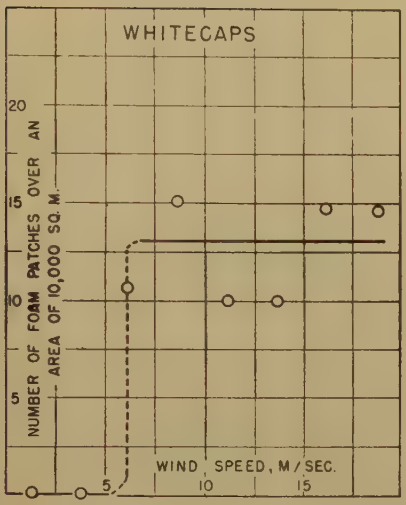


Figure 47. Foam patches are observed only for wind speeds exceding 7 m/sec.

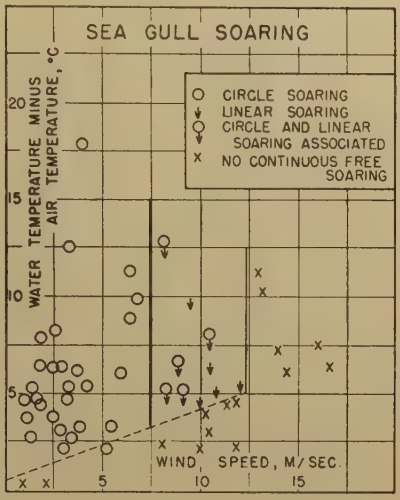


Figure 48. Observations of soaring sea gulls reveal a change in tactics at a wind speed of about 7 m/sec. This is believed to be the result of changes in the convection pattern above the sea surface. After Woodcock (1940).

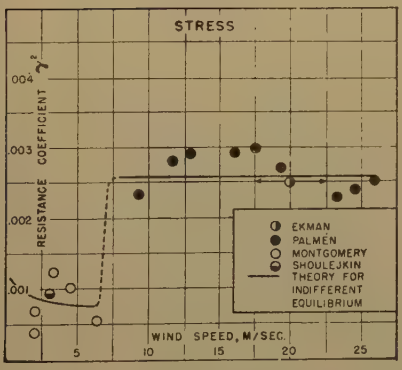


Figure 49. Observations of storm tides and of the vertical wind profile over the ocean reveal a sharp change in the resistance coefficient at a wind speed of about 7 m/sec. Arrows to both sides of Ekman's observation indicate range of wind speeds over which observation is applicable.

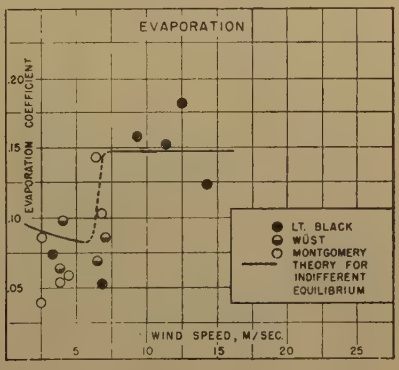


Figure 50. Observations of the humidity gradient above the sea indicate a sharp change in the evaporation coefficient at a wind speed of about 7 m/sec. After Sverdrup (1946).

birds move in a line directly against the wind flow." Observations of soaring during periods when sea temperature exceeded air temperature[4] have been plotted on Fig. 48; each of these 53 observations extended intermittently over an average time interval of six hours. For winds above 7 m/sec there is both circle-soaring and linear-soaring. According to Woodcock, "there is no linear soaring below 7 meters per second."

*Resistance Coefficient.* Taylor (1916) has defined a dimensionless parameter, the resistance coefficient $\gamma^2$, by the equation

$$\gamma^2 = \tau/\rho' U^2 , \tag{1}$$

where $\rho'$ is the density of air, $U$ the wind speed at an elevation of approximately 15 meters above sea level, and $\tau$ the mechanical stress exerted by the wind on the sea surface. Stress cannot be measured directly but may be computed from indirect evidence. Ekman (Sverdrup, *et al.*, 1942: 490) computed stress from measurements of sea level rise during a storm in the Baltic (storm tides). Palmén and Laurila (1938) made a similar study, dealing with the effect of wind on sea level in the Gulf of Bothnia, wind speeds ranging from 9 m/sec to 26 m/sec. Within this range $\gamma^2$ appears to have a constant value of about 0.0026 (Fig. 49), so that the stress equals

$$\tau = 2.6 \times 10^{-3} \rho' U^2 . \tag{2}$$

The implications of this relationship, in the light of similar "quadratic laws" for the flow of fluids over *rough* plates, have been discussed by Rossby and Montgomery (1935) and will be dealt with later. Further evidence in favour of a constant value of $\gamma^2$ for winds exceeding 6 to 8 m/sec can be found in the successful application of equation (2) to such oceanographic processes as upwelling along the coast of southern California (Sverdrup and Fleming, 1941), the generation of waves by wind action (Sverdrup and Munk, 1947), and the maintenance of equatorial currents by wind stress (Sverdrup, in press).

At low wind speeds the empirical data are much less conclusive. Rossby has found Von Kármán's equation for the flow of fluids over *smooth* plates in agreement with observations by Montgomery and by Shoulejkin of the wind profile above the sea (Rossby, 1936: fig. 1). Stress can be computed from simultaneous measurements of wind speed at two levels according to the equation

[4] In cases where the air temperature exceeds the water temperature, factors related to the stability in the lowest air layers tend to obscure the transition at the critical wind speed.

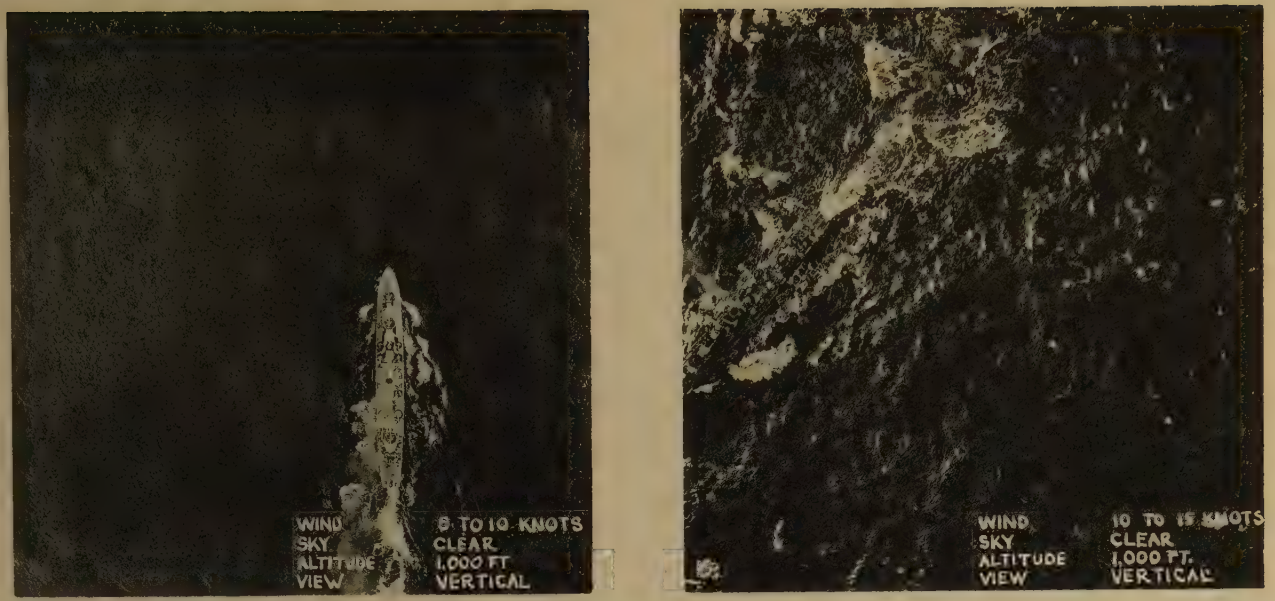


Plate III. Vertical view of the sea surface from an elevation of 1000 feet. The left photograph was taken during somewhat weaker winds than the right photograph, but otherwise under identical conditions. (Official Photograph, United States Navy.)

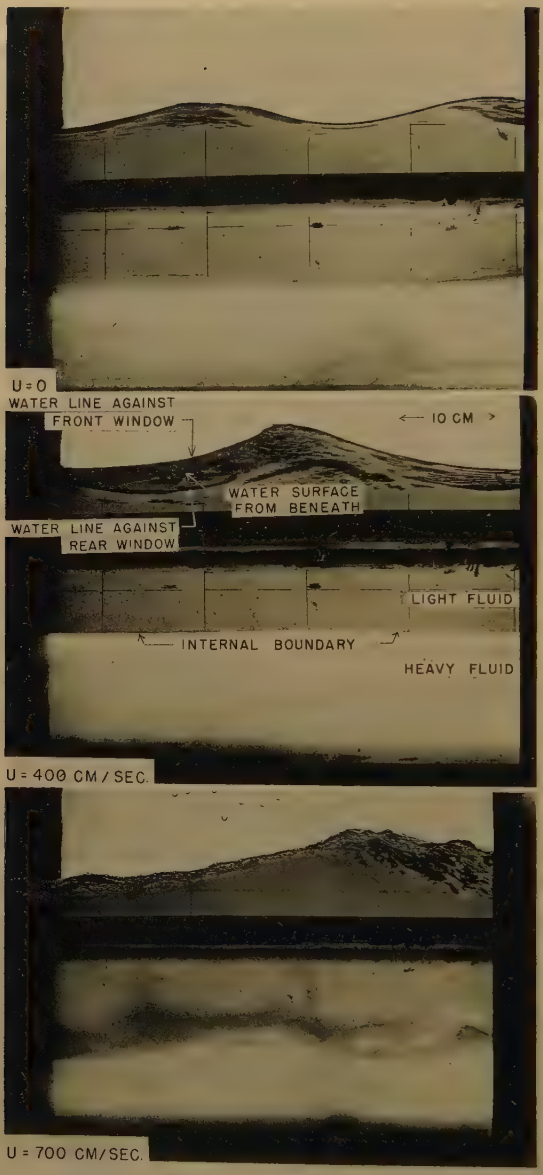


Plate IV. Effect of wind on waves generated mechanically in an experimental tank. The camera was beneath the level of the water surface outside the tank, so that the boundary is seen from below. For a wind of 700 cm/sec (bottom photograph) the water surface is rough and eddies form at the internal boundary.

$$\sqrt{\tau/\rho'} = \frac{k\,(U_2 - U_1)}{\ln\,(z_2/z_1)}, \tag{3}$$

where $k = 0.4$, $U_1$ and $U_2$ are the wind speeds at elevations $z_1$ and $z_2$, respectively. Values of the resistance coefficient for low wind speeds have been computed from these observations and plotted in Fig. 49. All wind speeds have been adjusted to an elevation of 15 meters by means of the equation

$$\frac{U_{15} - U_1}{\ln\,(z_{15} - z_1)} = \frac{U_2 - U_1}{\ln\,(z_2/z_1)}. \tag{4}$$

Montgomery's observations were made simultaneously at two levels, Shoulejkin's observations at four different levels. The solid curve segments in Fig. 49 refer to Rossby's equations for the stress over smooth and rough surfaces.

*Evaporation Coefficient.* Montgomery (1940) has defined another dimensionless parameter, the evaporation coefficient, by the equation

$$\Gamma = -\frac{1}{e_s - e_a}\frac{de}{d\ln z}, \tag{5}$$

where $e$ is the vapor pressure at an elevation $z$ above the sea surface, $e_s$ the vapor pressure at the sea surface, and $e_a$ the vapor pressure at a standard level above the sea surface.

Sverdrup (1946) has discussed the relationship between wind speed and the humidity gradient over the sea surface, and he finds that for indifferent equilibrium conditions the evaporation coefficient changes abruptly from a value of .08 to almost .15 at a wind speed of about 6 m/sec. His striking graph is reproduced in Fig. 50. The solid curve segments show the theoretical relationships for smooth and rough surfaces.

*Laboratory Measurements.* Plate IV shows three photographs taken during laboratory measurements conducted at the Institute of Hydraulic Research, University of Iowa, by Dr. H. Rouse and the author. Waves were generated by a plunger at one end of a flume and wind blown over the water surface.

The three photographs show the wave motion at gradually increasing wind speeds. By using two layers of fluid of different color and density, the water motion beneath the surface can be studied. No appreciable mixing across the internal boundary can be noted until the surface wave breaks; thereupon the laminar flow pattern is disturbed and eddies are formed at the internal boundary. Thus the effect of the

critical wind speed is not confined to the sea surface and to the layers directly above and beneath the surface, but appears to make itself felt at greater depths. The laboratory measurements are summarized in Table II.

TABLE II. Results of Laboratory Measurements

| | | |
|---|---|---|
| Mean wind speed, $U_{mean}$, in m/sec | Above 4.5 | Below 4.5 |
| Wind speed above crests, $U_{crest}$, in m/sec ($U_{crest} = 1.4\ U_{mean}$) | Above 6.3 | Below 6.3 |
| Number of runs during which crests broke | 10 | 1 |
| Number of runs during which no breaking occurred | 3 | 15 |

The wind speed $U_{mean}$ refers to a mean water level 14 cm beneath the top of the flume. The elevation of the average breaking crest above the mean level equalled 4 cm, so that the thickness of the air layer above the crests equalled 10 cm. According to Bernoulli's principle the product of the thickness of the air layer with the air speed must remain constant and the air speed directly above the crests, $U_{crest}$, must equal 1.4 $U_{mean}$. According to Table II, breaking occurred in 77% of the runs for which the wind speed above the crests exceeded 6.3 m/sec; and in 6% of the runs for which the wind speed was less than 6.3 m/sec.

*Other Observations.* Langmuir's (1938) careful measurements on Lake George imply a distinct change in the convection pattern at wind speeds somewhere between 4 and 8 m/sec for indifferent equilibrium conditions. A similar change in the convection pattern is reflected in the vertical distribution of temperature and salinity in the upper layer of the ocean (Armstrong, 1947). Another indication of discontinuity is the marked change in "sea return" on radar screens at a wind speed of about 7 m/sec. Also the general noise level in the sea and the reverberation from the sea surface are affected though not apparently to an equally marked degree.

The transition from a stable to an unstable sea surface is likely to have important bearing upon the capacity of the sea surface to reflect light and other radiation; it may affect still other processes. Observations should be planned and their analyses carried out to allow for a discontinuity at Beaufort 4.

## THE TURBULENCE MODEL BY ROSSBY AND MONTGOMERY

From wind observations over the ocean, Rossby and Montgomery (1935), Rossby (1936), have inferred the existence of two hydrodynamically distinct types of sea surfaces. In order to account for the

profile observed at low wind speeds, Rossby assumes a 'hydrodynamically smooth' surface with a very thin (about 1 mm) laminar boundary layer directly above it. For winds exceeding a certain critical value, Rossby and Montgomery assume the sea surface as 'hydrodynamically rough' with turbulence extending to the very surface. The authors do not state the critical wind speed explicitly, but their discussion implies a value somewhere between 6 and 8 m/sec at indifferent stratification.

In this manner Rossby has been able to account for the difference in stress at low and high wind speeds; indeed the data shown in Fig. 49 contains some of the evidence used to establish the concepts of smooth and rough sea surfaces. Sverdrup (1946) has used the Rossby-Montgomery model to account theoretically for the abrupt change in evaporation at a wind speed of about 7 m/sec (Fig. 50). Woodcock (1940) follows similar reasoning to explain the change in soaring tactics of sea gulls.[5] Also, it seems likely that white caps, breaking waves in tanks, convection patterns above and beneath the sea surface, radar "sea return," and noise level in the ocean, are all related (directly or indirectly) to the hydrodynamic character of the sea surface.

The laws applied by Rossby and Montgomery to the wind profile over the sea surface are identical with laws established in laboratory experiments dealing with the flow of fluids over plates. At low wind speeds air flow over the ocean is analogous to flow of fluids over smooth plates; at high wind speeds it is analogous to flow over plates which have been artificially roughened by means of sand grains. In view of these analogies the concepts of a hydrodynamically smooth and hydrodynamically rough sea surface were introduced. However, these laboratory experiments provide no basis for assuming that the sea surface can have the characteristics of *either* a rough or a smooth surface, depending upon the wind speed. A possible basis for assuming the curious dual role of the sea surface is presented in the next section.

## THE INSTABILITY CRITERION

Two fluids, of densities $\rho$ and $\rho'$, are separated by a sharp boundary (Fig. 51). The ratio $s$ of the upper density to the lower density is assumed to be less than one. Let $C$, $L$, and $H$ denote the speed,

[5] "There is some evidence that the force exerted by the wind on the sea surface cannot be expressed by a single continuous function of wind velocity and other factors, so that the critical values of these factors are also of interest to oceanographers. The soaring gulls seem to provide some clues as to the critical wind speeds at which there are marked changes in the form of convective motion, and thus in the transfer of momentum between the two mediums."

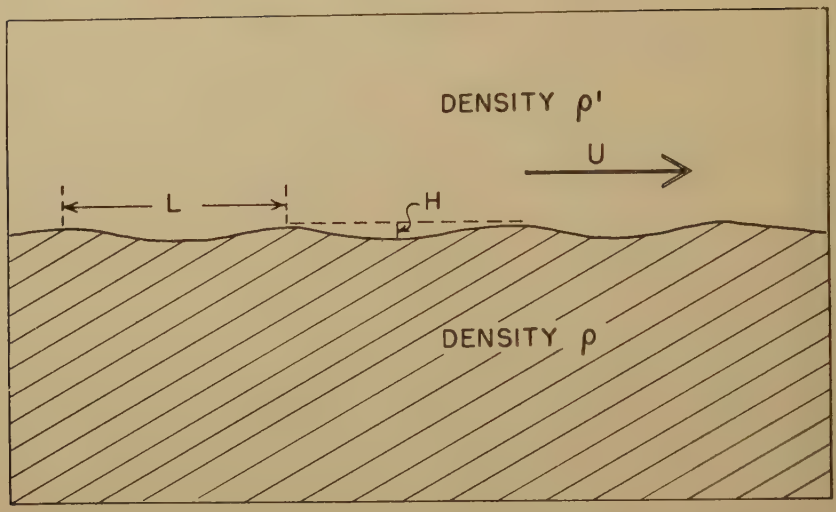


Figure 51. A fluid of density $\rho'$ is separated by a sharp boundary from a fluid of density $\rho$. The upper fluid moves relative to the lower fluid with a uniform speed $U$. Waves at the boundary between the fluids have a height $H$, a length $L$.

length and height of waves formed at the interface in the absence of currents. If the upper fluid is moving with a speed $U$ relative to the lower fluid, then to any particular value of the wave velocity $C$ there corresponds a critical fluid speed, $U = U'$ (Lamb, 1932; 373–374):

$$U' = C\left[\frac{1+s}{\sqrt{s}}\right]. \tag{6}$$

For $U < U'$ waves gradually diminish in height until they die out; for $U > U'$ waves grow according to the equation

$$H = \text{constant}\, e^{\alpha t}, \tag{7}$$

where

$$\alpha = \frac{2\pi}{L}\sqrt{\left[\frac{s}{(1+s)^2}\right]U^2 - C^2}, \tag{8}$$

until the height becomes so large that certain assumptions underlying the foregoing equations are transcended.

This instability criterion was established by Helmholtz (1868), who attempted to explain cyclonic disturbances at a surface separating cold and warm air. Applied to the air-sea boundary, the criterion shows that for any value of the wind speed, no matter how small, certain waves will grow, provided their velocity is sufficiently small.

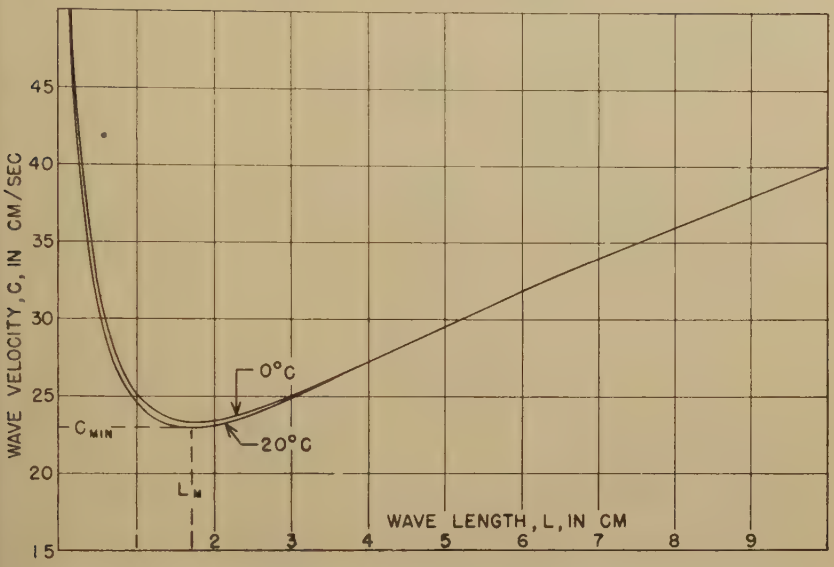


Figure 52. Wave velocity as function of wave length for two values of water temperature. The wave velocity has a minimum value, due to the opposing effects of gravity and surface tension.

Lord Kelvin (Thomson, 1871) has shown, however, that due to the effect of surface tension ($\sigma$), the wave velocity must always exceed a minimum value[6] as shown in Fig. 52:

$$C_{\min} = \left[\frac{1-s}{(1+s)^2}\frac{4g\sigma}{\rho}\right]^{1/4}. \tag{9}$$

The length of the waves of minimum velocity equals

$$L_m = \frac{2\pi}{\sqrt{1-s}}\sqrt{\frac{\sigma}{g\rho}}. \tag{10}$$

The general relationship between wave velocity and length becomes particularly simple in terms of the minimum values (Lamb, 1932: 459):

$$\frac{C}{C_{\min}} = \sqrt{\frac{1}{2}\left(\frac{L}{L_m}+\frac{L_m}{L}\right)}. \tag{11}$$

Fig. 52 shows the relationship between $C$ and $L$ for two extreme values of temperature.

[6] Lord Kelvin, during a boatride in the sound of Mull, which he undertook jointly with Helmholtz, was also the first to confirm the theoretically derived value for the minimum wave velocity.

Application of equation (6) to the special case of waves of minimum velocity leads to the definition of a critical wind speed, $U_c$,

$$U_c = C_{\min}\left[\frac{1+s}{\sqrt{s}}\right]. \qquad (12)$$

*For $U < U_c$ the sea surface is stable for waves of all lengths and periods.* Some numerical values are given in Table III.

TABLE III. Numerical Values of Minimum Wave Velocities and Critical Wind Speeds

$\rho' = 1.25 \times 10^{-3}$ g/cm$^3$
$\rho = 1.025$ g/cm$^3$
$s = 1.22 \times 10^{-3}$
Salinity $S = 33$‰

| Water temperature | 32° F (0° C) | 68° F (20° C) |
|---|---|---|
| Surface tension* $\sigma$ (g/sec$^2$) | 76.4 | 73.5 |
| $C_{\min}$ (cm/sec) (equation 9) | 23.3 | 23.0 |
| $L_m$ (cm) (equation 10) | 1.73 | 1.69 |
| $U_c$ (cm/sec) (equation 11) | 668.0 | 658.0 |
| $U_c$ (knots) | 13.0 | 12.8 |

* $\sigma = 75.64 - 0.144\ \vartheta$ (° C) $- 0.0221\ S$ (‰), (Sverdrup, *et al.*, 1942: 70).

The critical speed $U_c$ equals about 660 cm/sec, or 13 knots, and varies only slightly with temperature. This numerical value has been quoted in a number of investigations (Thomson, 1871; Jeffreys, 1925; Thorade, 1931: 42; Lamb, 1932: 462), but no further significance has been attached to it, as all these investigations were concerned with the lowest wind capable of raising waves. According to Jeffreys (1925) ". . . the velocity of a wind just strong enough to raise waves is actually only about 110 cm/sec . . .".

The derivation of the critical speed assumes a sharp break at the boundary. Although gusts from elevations of 15–30 feet may occasionally penetrate the lowest air layers and reach the sea surface without having lost a large fraction of their momentum, the *mean* wind speed undergoes a continuous, though a rapid, change in the region directly above the sea surface. Attempts to extend the Kelvin-Helmholtz analysis to a velocity profile which does not assume a sharp break at the boundary have met with great difficulties.

The profile of the mean wind and the amount of turbulence in the air depend upon the temperature gradient in the lower air layer. During typical summer conditions the water cools the air from beneath and the air is horizontally stratified. During winter the water is likely to be warmer than the air, so that convection and turbulence

stirs the lower air layers. Assuming the wind to be measured at a fixed elevation, it follows that instability of the sea surface is attained at lower wind speeds during the winter than during the summer. If whitecaps can be related to instability of the surface, this would explain the common observation that whitecaps are formed at lower wind speeds during the winter than during the summer. Compared to the effect on boundary stability of the temperature gradient in the air, any variations in surface tension or density due to the absolute variations in temperature are negligible (Table III).

## DISCUSSION AND CONCLUSIONS

A clear distinction must be made between wind waves and unstable wavelets. The dimensions of wind waves are many times the dimensions of the wavelets; furthermore, wind waves can be formed by winds of only 110 cm/sec, whereas wavelets require winds exceeding 660 cm/sec. During wind speeds between 110 and 660 cm/sec the sea surface is a hydrodynamically smooth surface, but it is not a *plane* surface. Small, smooth and stable wind waves are present. In addition, occasional gusts ruffle the surface, but the small corrugations are almost immediately smoothed under the action of surface tension. Once the wind speed exceeds the critical value, these corrugations not only persist, but also grow rapidly.

Direct observation of these wavelets is difficult because of their minute size. According to Table III their length at the critical wind speed is less than 2 cm. Careful observations indicate the existence of such very small corrugations; occasionally they appear to break and leave behind a tiny trail of bubbles. The wavelets, although too small to account directly for such conspicuous features as whitecaps, may be responsible for determining the characteristic roughness of the sea surface. Table IV gives the maximum dimensions of the wavelets according to equations (6) and (11), assuming the height of the wavelets to equal between 1/3 and 1/2 their length.

TABLE IV. Dimensions of Unstable Wavelets at Various Wind Speeds According to the Kelvin-Helmholtz Theory

| $U'$ (m/sec) | $L$ (cm) | $H$ (cm) |
|---|---|---|
| 6.61 | 1.71 | 0.7 |
| 10.0 | 7.5 | 3.0 |
| 15.0 | 17.7 | 7.0 |
| 20.0 | 31.4 | 12.6 |
| 30.0 | 71.0 | 28.4 |

According to Rossby (1936: 19), "the roughness parameter (of the sea surface) corresponding to steady moderate to strong winds seems to be in the vicinity of 0.6 cm. . . .," and within this range does not change markedly with wind speed. The average height of the unstable wavelets in the range of wind speeds between 10 and 20 m/sec is 11 cm, or roughly 20 times the roughness parameter. This factor lies intermediate between a factor of 30 suggested by Prandtl (1932) on the basis of wind tunnel experiments using sand grains for roughness and a factor of 10 determined by Sverdrup (1936) from measurements of wind over a snow field. Although the computed dimensions of the wavelets do not meet Rossby's requirement of being independent of wind speed, it may be significant that they are of the order of magnitude suggested by Rossby. In all events it is considered significant that *the Kelvin-Helmholtz theory gives instability at a wind speed which corresponds, within the limits of observational error, to the critical speed for transition from a hydrodynamically smooth to a hydrodynamically rough sea surface.*

The manner in which stability or instability bears upon turbulence and other processes at the air-sea boundary is only conjecturable, but so are any of the other causes of turbulence. Most of our knowledge is based upon the investigations by Rayleigh and Kelvin (Lamb, 1932: 670–674) into the stability of infinitely small disturbances superimposed upon flow through pipes and channels. In analogy with these investigations it may be reasoned that instability at the sea surface is the *cause* of turbulence at the boundary. From this point of view the Kelvin-Helmholtz criterion can be interpreted in terms of a nondimensional number, $U\sqrt{s}/C_{\min}\ (1 + s)$; a value of "one" marks the transition from laminar to turbulent flow at the boundary, just as a Reynolds number of 2000 marks the transition from laminar to turbulent flow in pipes.

The analogy may be carried further, and the pattern in the air flow over the wave crests compared to the flow of fluids around cylinders (Figs. 53, 54). As long as the flow is laminar the stream lines follow the entire wave profile, and the resultant force on the wave profile is zero (Lamb, 1932: 77). When irregular turbulent flow has set in the pressure against the wave profile is proportional to the square of the wind speed; the air flow spearates from the sea surface shortly to the lee of each crest and impinges upon the windward side of the next crest. A negative pressure or suction is likely to be found near the point of separation. An eddy is formed to the lee of each crest. The roughening of the sea surface permits the wind to grip the water and to apply greater tangential stress.

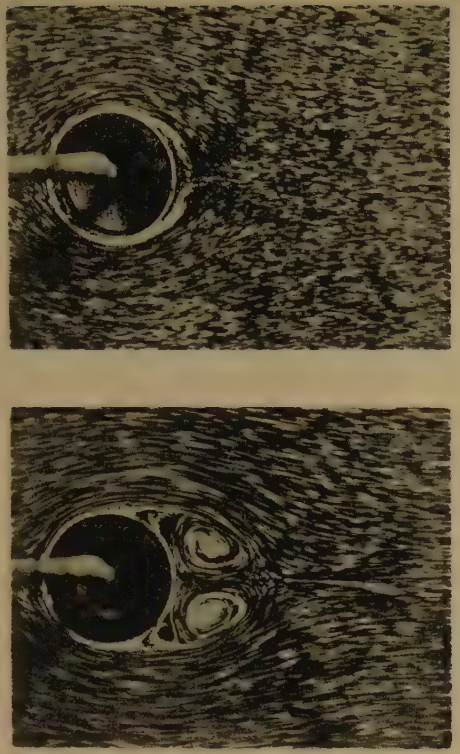

Figure 53. Observed flow of fluid past a cylindrical obstacle. Top figure: low speed; bottom figure: high speed. After Goldstein (1938: I 63).

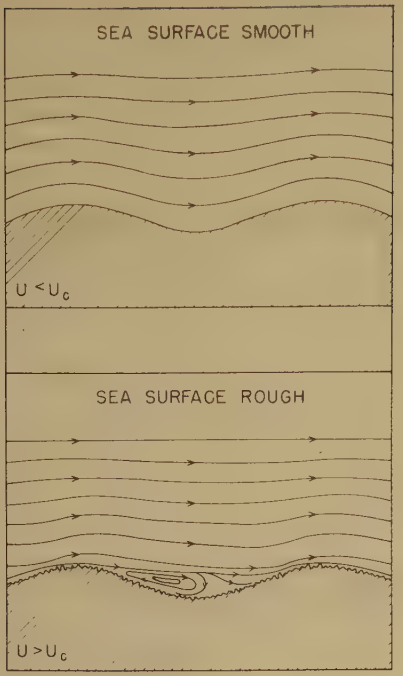


Figure 54. Assumed flow of air over waves. Top figure: less than critical speed; bottom figure: greater than critical speed.

The growth of surface waves, due to the combined action of wind pressure against the wave profile and tangential stress along the sea surface (Sverdrup and Munk, 1947: 9–12), would be accelerated by all these factors. Presumably this acceleration accounts for incipient whitecaps at wind speeds slightly in excess of the critical speed. Therefore the appearance of whitecaps is interpreted as the most direct visual evidence of the transition to instability. It is not difficult to see that other boundary processes would be affected materially at this point.

Two related problems, one the breaking of waves in the open sea during winds much higher than the critical speed, the other dealing with the initial formation of waves during winds much less than the critical speed, go beyond the scope of this paper and hence can be mentioned only briefly. The first problem concerns interference between wave trains; in that connection Unna (1941) has pointed out that the converging flow pattern in front of the crests of waves must lead to an effective steepening of small waves overriding the main wave system. The second problem has been treated by Jeffreys (1925) on the premise that for a wind speed of 110 cm/sec (the lowest wind capable of raising waves) the rate at which energy is supplied by normal wind pressure must equal the rate at which energy is dissipated by molecular viscosity. In assuming the normal pressure proportional to the square of the wind speed, Jeffreys implies fully developed turbulence at winds of 110 cm/sec, in contrast to the evidence presented in this report. Furthermore, Jeffreys obtains a value for the "sheltering coefficient" which is more than twenty times the value obtained by Sverdrup and Munk (1947: 22) from observations of the rate of growth of wind waves. The problem of initial wave formation therefore deserves further consideration.

The author is indebted to Dr. H. U. Sverdrup, Mr. R. S. Arthur and Mr. D. F. Leipper for suggestions and criticism.

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# A FISH FARMING EXPERIMENT IN SCOTTISH SEA LOCHS[1]

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The practice of stimulating dense growths of marine phytoplankton in laboratory cultures by the addition of nitrate and phosphate is now well established. Since in the sea the food chain leading ultimately to fish begins with inorganic nutrient salts, and passes through plankton and bottom fauna, it seemed feasible to attempt to stimulate the food cycle by the addition of artificial fertilizers to seawater areas. Nitrate and phosphate alone were considered since these are the nutrients in short supply in the sea.

The work was carried out by Dr. F. Gross and the author, working from the Zoology Department, Edinburgh University, with the collaboration of Drs. S. M. Marshall and A. P. Orr of Millport, and later of two research workers, Mr. Nutman and Dr. Gauld. The cost of the work was generously defrayed by Imperial Chemical Industries, Ltd., Billingham, England.

## THE LOCH CRAIGLIN EXPERIMENT

In order to obviate as far as possible a great loss of fertilizer, experiments were commenced on a very small, much enclosed, sea-loch (Loch Craiglin), about eighteen acres in extent. The loch is comparatively shallow and has a very narrow channel open to the sea, which was dammed off, so as to restrict fertilizer loss. Loch Craiglin, however, had relatively few features to recommend it apart from its small size. We could not admit outside sea water regularly to the loch, as we had planned, and as a result there were great fluctuations in hydrographic conditions:—variations in salinity particularly at the surface; very high pH in the upper water layers; low pH and oxygen content and some $H_2S$ production in deeper waters due to decomposition, especially of seaweed.

The distribution of fertilizer consisted in the application throughout 1942 and 1943 of some twelve hundred pounds of sodium nitrate and

[1] One of a series of papers presented at a symposium, "Fertilization of Aquatic Areas," sponsored by the Limnological Society of America. Boston, 1946.

six hundred pounds of superphosphate. This mixture was distributed usually as a powder at intervals throughout spring, autumn and winter, but no fertilizers were added in summer as they encouraged at that time an extremely heavy growth of seaweed and *Zostera*. The quantities of fertilizers added were calculated so as to obtain a concentration of 100 mg/m$^3$ of nitrate and 20 mg/m$^3$ of phosphate throughout the water immediately after fertilization.

## Results

*Plankton.* Most unexpected was the rapidity with which the fertilizer was utilized. During the ordinary growth season the bulk of that added was completely used within a week. Of the various categories of phytoplankton organisms, the very small flagellates (from 1 to 8 μ in diameter), or nannoplankton as they are termed, showed the most striking response. On several occasions their numbers rose immediately after fertilization and at times there were tremendous concentrations of up to 8,000 per cu. mm.—that is, 8,000,000 per cc. The effect on diatoms throughout 1942 and 1943 was less obvious, but these also increased at times to very high densities (7,000 cells per cc.); dinoflagellates were also fairly abundant throughout that period of the work.

It is a remarkable fact that the heaviest growth of phytoplankton for 1942–1944 was experienced in winter, early in 1944, with maxima of 8,000 peridinians, 52,000 diatoms and 6,000,000 μflagellates per cc. of water. During this period the water was to the eye green or brown according to the dominating form. Very striking responses to fertilization of diatoms and dinoflagellates, as opposed to μflagellates, were found in 1945.

As regards the zooplankton, in the spring of 1943 the population in Loch Craiglin reached a very high density, but afterwards there was no response, although the phytoplankton was very high. The limitation in the zooplankton production is considered to have been due to hydrographic conditions—particularly high $H_2S$ concentration in deeper layers, high pH in upper layers, and possibly the much reduced salinity.

*Bottom Fauna.* The bottom fauna changes were followed by grab samples taken at fixed stations before fertilization began and throughout the course of the work. This method of control was found to be much superior to selecting an outside control area, since the bottom fauna of Loch Craiglin was very specialised. The response of the bottom fauna was not obvious during the first year of the experiment, but by the summer of 1943 it had reached an average density over the

whole loch 240% greater than that of the summer before (1942). Comparing the average *winter* densities in Loch Craiglin, by January 1944 the average density was 300% greater than in the previous winter. An average population of 15,000–16,000 animals per sq. m. throughout the summer and early autumn of 1943 is especially remarkable in view of the fact that the figures represent a dynamic balance between the number of bottom animals reproduced and the amount grazed down by feeding fish. Some 3,000 flatfish were transferred during 1942 to Loch Craiglin and about 23,000 in 1943, in addition to a considerable native population of fishes, so this grazing effect was very considerable.

*Fish Growth.* Although the samples of fish obtained were on the whole disappointingly small, the growth rates were fortunately fairly consistent.

About 170 plaice of under 9 cm. length were transferred to Loch Craiglin. They grew about 12 cm. in a year, as compared with annual increments of 3 to 8 cm. by plaice of the same age on various natural grounds.

Two thousand flounders under a year old (*i. e.*, Age Gp.—0), transferred in 1942 (Stock I), had reached an average length of 12 cm. at the end of their first year in Loch Craiglin. This growth may be compared with flounders of the same age trawled from the natural area from which they were taken. On the natural ground the flounders grew in one year to an average length of only 5.5 cm. In 21 months, Stock I completed an amount of growth equivalent to five to six years of "normal" growth on various natural grounds. The growth of these flounders is three to four times faster by length, and between 16 and 19 times faster by weight.

One thousand flounders, between one and two years old (*i. e.*, Age Gp.—I) were transferred to Craiglin in the spring of 1943 (Stock II) in order to test whether or not the acceleration in growth rate applied only to the youngest fish. Stock II flounders completed about two years' normal growth in approximately six months.

A further 22,000 flounders (Age Gp.—0) were transferred in the summer of 1943 (Stock III). These showed considerable improvement in growth as compared with their controls, but the acceleration was not nearly as great as that of Stock I during the first year. This lesser growth rate cannot have been due to any food scarcity, since the bottom fauna had remained exceedingly rich. More likely the slowing down of growth was caused by increasingly unfavourable hydrographic conditions, especially the high concentration of $H_2S$ over the bottom late in 1943.

To test this hypothesis, during 1944 and 1945 the dam sluice was opened for long periods to allow considerable water exchange with the outside seawater. The growth of Stock III flounders showed a very marked acceleration after this, and in fact the increment of 13 cm. from June 1945 to March 1946 (9 months) is the largest so far recorded.

As it appeared that Loch Craiglin was rather unfavourable as regards hydrographic conditions, fertilization was begun in Kyle Scotnish, another arm of Loch Sween, which is larger and deeper than Craiglin. It must be appreciated, that in one respect the experiment in Kyle Scotnish was entirely new; in Kyle Scotnish no dam was erected, but the loch was left freely open to the outer parts of Loch Sween in order to test the possibilities of fish cultivation in *unenclosed* areas of the sea.

## The Kyle Scotnish Experiment

Kyle Scotnish is about 160 acres in extent. Near its connection with the main Loch and for a distance of several hundred yards it is very narrow, at one place only about 100 yards wide (the Narrows region). Further up, however, it widens out into two large basins (the South and North Basins). The maximum depth is about 20 metres in the South Basin, but the greater part of the Loch varies between about six and ten metres.

Fertilization began in January 1944; 2½ tons of fertilizer were distributed usually every month, which means about 3 cwts. per acre per annum. Superphosphate was always used as phosphate fertilizer, but as regards nitrogen, we began with sodium nitrate and later changed to ammonium sulphate. The fertilizers were raked dry into the water from a tray carried by a small motor boat.

One of the main problems was to estimate the amount of water exchanged between Kyle Scotnish and the outside. After some preliminary experimenting the phosphate concentration itself was used as an indicator of water movements. In order to do this, on several occasions the fertilizers were distributed only in the North Basin of Kyle Scotnish so as to obtain there a relatively very high concentration of nutrients, initially confined to a small area. We studied the subsequent extent and rate of dispersal of the phosphate from the North Basin. For several fertilizations, during the first 24 hours after the phosphate was added, it did not become dispersed beyond the North Basin, but during the second and third days after fertilizing it spread to the South Basin. Phosphate did not reach the Narrows region till the third day and water samples from the mouth of Kyle Scotnish showed phosphate only in the same low concentration as was

found outside right up to approximately a week after fertilizing, by which time the bulk of the phosphate added had been used.

The rapidity of this utilization (the same as was found in the Loch Craiglin experiment) is one of the most important features of the fertilization work. Only approximately one quarter of the phosphate added dissolved immediately at fertilization; the rest of the fertilizer presumably sank to the bottom. But a considerable amount of the phosphate going to the bottom subsequently went into solution and became available to the phytoplankton.

Even with adverse winds and tides, as far as our experiments went, the loss of fertilizer was never sufficiently great to cause anxiety. It must be appreciated that water masses are relatively stable and mix comparatively slowly.

## The Effects of Fertilization

*Phytoplankton.* An outside arm of Loch Sween (Sailean More) was used as a control area for both phytoplankton and zooplankton, and it should be emphasized that before fertilization began the plankton populations of Kyle Scotnish and of Sailean More were very similar both as regards species and density.

μflagellates occurred in Kyle Scotnish generally in lesser numbers than we found them in Loch Craiglin. In Scotnish it was the larger members of the phytoplankton which were usually more important. Perhaps some sort of competition exists between the μflagellates on the one hand and the diatoms and dinoflagellates on the other, for the fertilizer added. Even so μflagellates occurred in Scotnish in numbers up to 1,500 per cu. mm., so that their contribution to the general organic production is in no sense to be disregarded, and they are probably of great importance as a food supply for small larvae.

As regards the diatom and dinoflagellate populations, at times this phytoplankton responded very favourably to fertilizer application, for instance in April 1946 when in five days the phytoplankton of Kyle Scotnish rose to between two and three times the population in Sailean More. Similar increases followed fertilizer distribution in May, and June, when the population was four times as high in Scotnish as outside.

These results would appear to be highly satisfactory, but on other occasions there was *apparently* little if any response from the phytoplankton to fertilizer distribution. This was particularly true during summer, when actually the phytoplankton of Sailean More was sometimes even richer.

The reason for this apparent lack of response in phytoplankton in Scotnish was, we believe, the tremendous grazing activities of the zooplankton.

*Zooplankton.* Before fertilization began in Kyle Scotnish there was no significant difference between the zooplankton populations in Kyle Scotnish and in Sailean More. But from March 1944 the zooplankton of Kyle Scotnish became steadily more numerous. In May there were, on the average, 10 animals per litre in Sailean More, as compared with between 30 and 60 per litre in different parts of Scotnish. In July there were 50 in Sailean More, as compared with over 200 in the South Basin of Scotnish and over 370 in the North Basin.

Throughout most of the year, both in 1944 and 1945, the zooplankton density was considerably greater in Kyle Scotnish than in Sailean More, and on those few occasions when the zooplankton of Sailean More was richer, the populations were very low in both areas.

In Loch Craiglin, during the comparatively few periods when the zooplankton was rich, the phytoplankton crop remained relatively very small in spite of the addition of fertilizers. This phenomenon was much more common in Kyle Scotnish where the conditions for the growth of a rich and varied zooplankton population were more favourable. It seemed that dense zooplankton must keep down the phytoplankton production.

On those occasions when zooplankton in Kyle Scotnish was poor and, it is claimed, conditions for phytoplankton growth were therefore favourable, the rise of phytoplankton population, following the addition of fertilizer, was very great. Thus, during October 1945, when zooplankton was rather low, the population of diatoms and dinoflagellates together in Kyle Scotnish rose from 700, before fertilization, to 3,800 a week after—an increase of more then five times. As compared with this, the phytoplankton population in the control area showed a slight decline.

*Bottom Fauna.* The bottom fauna was analysed, as in Craiglin, by grab samples taken at fixed stations in the loch, both before fertilization and throughout the following two year period.

On the whole the muddy areas of the North and South Basins were comparatively poor both in species and in actual densities, and not every station showed an increase after fertilization was started. If the populations during the winter (January) of 1945 and 1946 be compared with the populations at the same stations in January 1944 (*i. e.*, before fertilization started), of altogether 27 stations, 14 showed an increase in both years of fertilization, while a further six stations showed a decline in one year (mostly in 1945) and then an increase in the second year of the work (1946). The percentage changes show that some stations experienced very striking increases, such as in the Narrows stations (up to 700%), in the South Basin (up to 250%), and

in the North Basin (up to 600%). Only a single station showed a striking decline.

The average bottom population for the whole of Kyle Scotnish, calculated from the various densities at each station, shows a steady rise throughout the two years to three and four times the prefertilization value. Table I shows that this steady increase over the two

TABLE I. Effect of Fertilization on the Bottom Fauna of Kyle Scotnish. Numbers Represent the Average Population of Bottom Fauna per Square Metre, Calculated from Densities at Individual Stations

| | *No. of Stations* | *Pre-fertilization 1943–44* | *Post-fertilization 1944–45* | *1945–46* |
|---|---|---|---|---|
| SUMMER (Grab Series) | 6 | 640 | 1000 | 1950 |
| WINTER (Grab Transects) | 23 | 460 | 850 | 1420 |
| WINTER (Regular Stations) | 7 | 680 | 1400 | 3160 |

fertilization years holds, whether summer or winter samplings be considered.

The most striking change in the bottom fauna on shore between tide levels was the tremendous settlement of the common mussel which took place over the only extensive area of shore in the Loch. Table II

TABLE II. Effect of Fertilization on Mussel Settlement, Summer, 1945. All Figures Represent Number of Living Mussels per Square Metre. Two Different Areas Were Sampled in Both Kyle Scotnish and Linne Mhurich; One Area Only in Sailean More

| | | *<10 mm* | *10–40 mm* | *>40 mm* |
|---|---|---|---|---|
| KYLE SCOTNISH (Fertilized) | Area 1 | 9,300 | 4,300 | ca. 10 |
| | Area 2 | 396,000 | 556 | 136 |
| LINNE MHURICH (Unfertilized) | Area 1 | 0 | 160 | 184 |
| | Area 2 | 32 | 8 | 72 |
| SAILEAN MORE (Unfertilized) | | 9,712 | 2,088 | 1,068 |

shows the total density per square metre of young mussels in Scotnish during the summer of 1945, and comparative figures are also given for unfertilized arms of Loch Sween. These figures illustrate the extraordinary high density of settlement in Scotnish, and this despite the fact that the area is a very soft mud, highly unsuitable for mussel settlement. This suggestion of the unsuitability of Scotnish shore is borne out by the fact that the breeding stock of large mussels is found in much greater numbers in the outer arms of Loch Sween than in Kyle Scotnish. For instance, there existed in Kyle Scotnish in the summer of 1945 between 900 and 3,000 young mussels for every large

one, whereas in two outside areas there were less than 10 young spat for every adult (Table II).

The average population of the shore, apart from the mussels, consisting of *Hydrobia*, chironomid larvae, polychaetes, bivalves, etc., also rose from November 1943 to November 1945 by 40%, the increase affecting practically all the individual species.

*Fish Growth.* Apart from some flounders and a few plaice, Kyle Scotnish possessed a very sparse natural fish population. It was obvious that the area would have to be stocked just as in the case of Loch Craiglin.

In studying the growth, both of flounders and plaice, throughout the work, Dr. F. Gross removed otoliths from the fishes to verify their age determinations.

In the spring of 1944 only a comparatively few artificially fertilized eggs of plaice, cod and haddock were introduced. In the spring of 1945, however, stocking with 2,000,000 plaice eggs and fry from a new hatchery was achieved. These Gp.—O plaice were then studied by means of successive samplings by seine-net fishing in the Loch, and although the samples were sometimes small, the mean sizes of the successive samplings obtained showed clearly and conclusively a very satisfactory growth. In their first season these Gp.—O plaice grew to 13 cm. and reached 25 g. in weight. This represents five times the normal weight increment. Furthermore, there were some Group—I plaice from the previous year that came into the samplings. They reached a size by September 1945 (when they were 1½ years old) of 20 cm. and 100 g.—approximately three times heavier than naturally good growing fish of the same age.

The natural population of flounders in Kyle Scotnish was sufficient to yield some good confirmatory data on the beneficial effects of fertilizer on fish growth. Group—O flounders grew to about 10 cm. in their first year and weighed 10 g. This is an improvement by about 400% in weight over the growth on normal grounds. Group—I flounders reached an average size of over 20 cm. and a weight of nearly 120 g. This is a great improvement on normal conditions and in fact approaches the best growth observed in the earlier experiment in Loch Craiglin.

Even these figures, satisfactory as they are, underestimate the improvement in growth in Scotnish under conditions of fertilization. For we have considered only the mean sizes of the various groups of fish, but the means are not a true index of growth rate. Owing to the migration of fish to and from the fertilized loch the size range of any one Age Group becomes unusually large and the size-distribution

curves become skewed. A careful study of successive growth curves showed further that the migration was not even a random movement, but that small specimens migrated in from outside and the larger specimens (older inhabitants of Scotnish) moved out of the Loch. It is well known that as flatfish grow up, apart from any random dispersal, they tend to move offshore. As their growth rate is increased their departure to offshore areas is presumably accelerated, but this reduces the improvement in growth that can be shown by means of distribution curves.

This implies that fertilization of an open sea loch such as we have been practising will benefit flatfish only during the first year or two of their lives; after that they tend to migrate outwards. For similar reasons no data were obtained on the growth rate of round fish.

## SUMMARY

In both enclosed and unenclosed small sea areas, the addition of nitrate and phosphate fertilizer stimulates phytoplankton growth, which in turn can maintain a high density of zooplankton and bottom fauna, and this in its turn leads to a great acceleration in the growth of flatfish. In two years, flatfish attained the equivalent of approximately four or five years' growth under normal conditions; and in fact they reach what might be called marketable size at the end of the two-year period.

Owing to the offshore migrations of flatfish, any further development should be carried out in an area with a natural fairly rich fish population, and this area must include waters of considerable depth so that fish can migrate there and still benefit from fertilization.

# MEASUREMENTS OF PRODUCTION IN FERTILIZED SALT-WATER[1, 2]

By

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The present paper describes some results of several exploratory experiments made at the Woods Hole Oceanographic Institution in connection with a proposed study of organic production in salt waters. An attempt is made to relate the viewpoint of the present paper to the more outstanding aims of fertilization work. Aside from practical matters of aquiculture, addition of nutrients to water is a valuable tool for studying what has been called food-chain dynamics. Briefly stated, the object of the described experiments was to determine immediate effects of fertilization upon the vital activities of the phytoplankton.

Of the four experiments to be mentioned, two were performed in an outdoor concrete tank built in Woods Hole for the purpose, the others in Rand's Harbor near Woods Hole. The tank at Woods Hole is made of high quality concrete and it is divided down the middle by a concrete partition, making two separate compartments, each being 5.70 x 2.74 x 1.53 meters. Notches in the wall permit insertion of heavy wooden partitions which divide each compartment into four equal sections. For purposes of these experiments, one undivided compartment was used. The concrete was thoroughly washed after setting and was leached with sea water for several weeks before use. No untoward effects of the concrete on the water were observed during the experiments; *e. g.*, the pH did not change during soaking with sea water after the initial washing, although it increased as expected during heavy plankton growth.

Rand's Harbor is an irregularly Y-shaped embayment of Buzzards Bay about ten miles from Woods Hole. The length of the larger branch is about 375 meters, of the smaller branch 250 meters, and of the foot roughly 125 meters. The end of each branch away from the seaward connection is a somewhat enlarged basin. The deepest part of the small branch is about 3.5 meters at high tide and of the large

[1] Contribution No. 414 from the Woods Hole Oceanographic Institution.

[2] One of a series of papers presented at a symposium, "Fertilization of Aquatic Areas," sponsored by the Limnological Society of America at Boston, 1946.

branch about 5 meters. The extreme range of tide is about 1.5 meters. The opening to Buzzards Bay is about 5 meters wide and 1.8 meters deep at high tide. Rand's Harbor was under intensive investigation between March 1 and September 1, 1946. On two occasions fertilizer was added to the smaller arm.

The work discussed here is part of a co-operative project initiated and supervised by Dr. George L. Clarke. The counts of tank plankton were made by Miss Mary Goffin, of the Rand's Harbor plankton by Dr. S. P. Chu. Miss Marion Hawes furnished much assistance in making chemical determinations. Other help is acknowledged subsequently.

## METHODS

*Oxygen.* Oxygen was determined by the Winkler method as described by Theroux (1943). The concentration of oxygen is expressed as milligram-atoms per liter as recommended by the International Association of Physical Oceanography (Sverdrup, *at al.*, 1942: table 34). Since oxygen is almost invariably given by limnologists as milligrams per liter, this mode of expression is also recorded in order to make values more readily comprehensible to all readers. The definite advantage of the former method is that the figure is proportional to the number of oxygen atoms rather than to the mass, which makes intercomparison of amounts of various elements easier in the light of definite chemical reactions and compounds. This is particularly useful with oxygen when considering photosynthetic equations and the amounts of carbon dioxide and carbohydrates involved. One mg. of $O_2$ = 0.0625 mg.-at.; one mg.-at. is 16 mg.

*Photosynthetic Oxygen Production.* The technique for measuring rate of photosynthesis is familiar, having been used by Manning and Juday (1941), Juday, Blair and Wilda (1943), Riley (1943), and others. In the present work, round clear glass bottles, with a volume of about 200 ml., were filled from the tank with raw water that was strained through stramin in order to remove large pieces of debris which might have been present. For each experiment four bottles were so filled, two of which were wrapped with several layers of black cloth and two left uncovered;[3] all four bottles were suspended in the middle of the tank at mid-depth in such a way that they were exposed freely to the light. Initial oxygen, phosphate, and plankton concentration were measured. At the end of two or four days the bottles were retrieved. Oxygen was determined on one pair of bottles, light and dark, phosphate and plankton on the other pair. The mean rates of change of

[3] Hereafter referred to as "dark" and "light" bottles.

oxygen and phosphate were then calculated by dividing the change from initial conditions by the time of exposure. The rate of increase of oxygen in the light bottle represents the rate of formation of oxygen by photosynthesis less the rate of consumption by respiration. The latter is measured by the change in the dark bottle. Subtracting the rate in the dark bottle (a negative quantity) from the rate in the light bottle gives the total mean rate of production.

The results of such determinations may be distorted by a number of factors which partially counterbalance each other. The phytoplankton started dying in the dark bottles while increasing in the light in these experiments. It might be thought, therefore, that the dark bottles did not sufficiently estimate the rate of respiration; however, it is known that bacteria and nonphotosynthetic protozoa multiply more rapidly in the dark bottles than in the light, and their respiration makes up for the loss of plankton, at least in part. The errors may be partly minimized by keeping the time of exposure as short as possible; yet the time must be long enough that the change in oxygen concentration is significantly larger than the error of determination. It is also necessary that the time of exposure of the bottle be short enough that the supply of carbon dioxide is not depleted. In these experiments, using sea water, this is unlikely to happen. Concordant results were given by three determinations, two covering two days each, which coincided with a longer four-day determination.

*Hydrogen-ion Activity.* The pH was determined with a Hellige colorimeter and the results corrected for salt error.

*Phosphate.* Phosphate was determined by the Atkins-Denigès molybdate method except that 1 ml. of solution was used instead of 2 ml. (Wattenberg, 1937). The intensity of the blue color was measured with a Klett photoelectric colorimeter which had been standardized with solutions of known phosphate content. Klett no. 66 filter and a 4 cm. cell were used. All measurements were corrected for salinity. Since the intensity of the blue color changes with time, all determinations were made during the period between five and fifteen minutes after the addition of reagents. The concentration is expressed as microgram-atoms of phosphate phosphorus per liter as recommended by The International Association of Physical Oceanography (Sverdrup, *et al.*, 1942: table 34). One μg.-at. of phosphate phosphorus is equivalent to 30.98 μg. of phosphate phosphorus, or 94.95 μg. of phosphate.

*Chlorophyll.* Two-liter samples of sea water were filtered in a modified Seitz filter through Whatman no. 42 paper on which had been deposited a layer of barium sulfate as described by Redfield, Smith and

Ketchum (1937: 423). One liter of water was used when much plankton was present. The entire filter medium was then ground with Merck reagent sea sand in a mortar and extracted with 80% acetone in the dark for fifteen minutes. The extraction mixture was filtered through a sintered glass filter and the filtrate read with a Klett photoelectric colorimeter.[4]

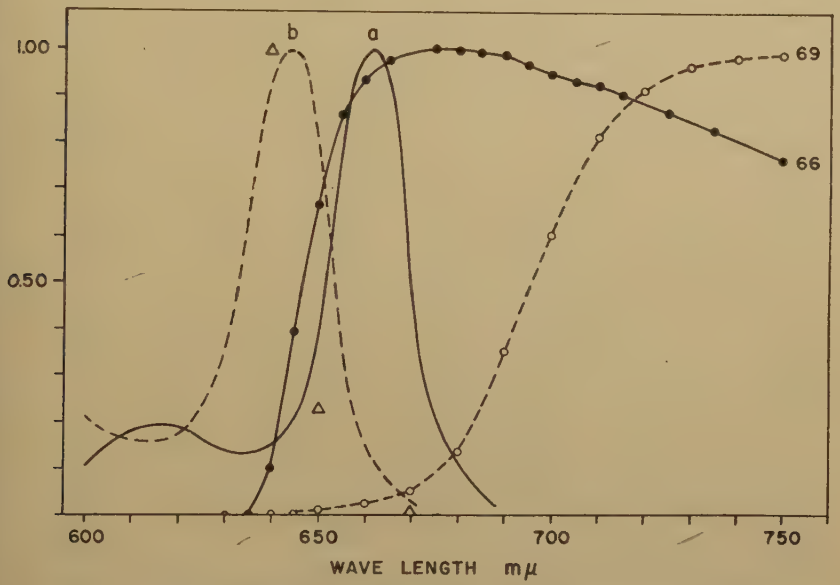


Figure 55. Absorption of light by chlorophylls *a* and *b* and chlorofucine (triangles) in acetone and transmission of light by Klett filters no. 66 and no. 69. For purposes of comparison, maximum transmission or absorption has been taken as 1.0 with all other values expressed as fractions of the maximum. Data for chlorophylls *a* and *b*. from Harris and Zscheile (1943), are relative absorptions ( = log $I_0/I$). Data for chlorofucine from Strain and Manning (1942). Data for transmission by the filters are simply the percentages of peak transmission at the given wavelengths.

Readings were always taken with Klett filters no. 66 and no. 69, since at the time it was uncertain which was most suitable. Subsequently the authors determined the transmission spectrum of the filters using a Beckman spectrophotometer. The results of these measurements are compared with the absorption spectra of chlorophylls *a* and *b* in Fig. 55. For the filters, the maximum percentage transmission was taken as 1, and the transmissions at various wavelengths calculated as fractions of the maximum. The curves for chlo-

[4] The instrument had been calibrated by Dr. Harold Haskin, using samples of pure chlorophylls *a* and *b* kindly supplied by Dr. Richard Goodwin of Connecticut College.

rophylls are derived from graphs published by Harris and Zscheile (1943). The maximum relative absorption in the red was similarly taken as 1 and relative absorptions at the various wavelengths expressed as fractions of the maximum. The graph indicates that Filter no. 66 is the better for measuring concentration of chlorophyll *a*, although there could be some interference from chlorophyll *b*. The calibration data showed that Filter no. 69 was relatively insensitive to both chlorophylls. There is no chlorophyll *b* in the diatoms which made up most of the plankton in this work. Chlorofucin may also interfere to some extent, but it is felt that this is a minor effect. Chlorofucin evidently is a normal component of diatoms (Strain and Manning, 1938).

*Plankton.* A 100 ml. sample of water was treated with six drops of Lugol's solution and allowed to settle overnight. The supernatant water was carefully poured off. The numbers of organisms were recorded, usually to genera, although in some cases species could be recorded and in other cases only families. The counting was done by the method described by Chu (1942). The period of settling should be much longer when small flagellates are present, but with the plankton present in the tanks, this period gave a sufficiently accurate estimation of the important elements of the phytoplankton.

In order to estimate somewhat better the actual amount of material present, the volume of each type of organism was estimated from mean size and the total volume of phytoplankton present calculated. Much reliance should not be placed on these volumes, since the mean sizes used are not precise.

## RESULTS

### Experiment 1

Only one of the experiments is to be described at length here. On the morning of August 20, 1946, the concrete tank at Woods Hole was filled with 15,000 l. of sea water pumped in from the adjacent harbor. The water was passed through a very large net of no. 10 bolting silk to remove the copepods and fish, reducing the larger zooplankton to a negligible number. Bottles were immediately filled for photosynthesis determination, using the raw sea water. Thereupon 100.5 g. of Reagent $NaNO_3$ dissolved in a small quantity of distilled water was poured into the tank and the water vigorously stirred to assure complete mixing. A second set of bottles was filled for another photosynthesis determination, using this nitrate fertilized water. Finally, 10.2 g. of Reagent $KH_2PO_4$ was similarly added, the tank stirred, and another set of bottles filled. Thus three determinations were run

concurrently for four days: one without fertilizer to serve as control (Fig. 56a), one with nitrate (Fig. 56*b*), and one with both nitrate and phosphate (Fig. 56*c*). Since only one tank was available at the time, a full control (a tank full of unfertilized water) was not possible. The amount of phosphate added was calculated to increase the concentration by 5.0 mg.-at. of phosphorus per liter. The initial concentration in the water was 0.93, and that measured after fertilization 5.82. The nitrate added was calculated to make the proportion of nitrogen to phosphorous 7.1 by weight or 15.6 by number of atoms. This is the average ratio in marine plankton (Sverdrup, *et al.*, 1942: 236).

Each day during the thirteen day period of the experiment, after the water was mixed by stirring, measurements were made of phos-

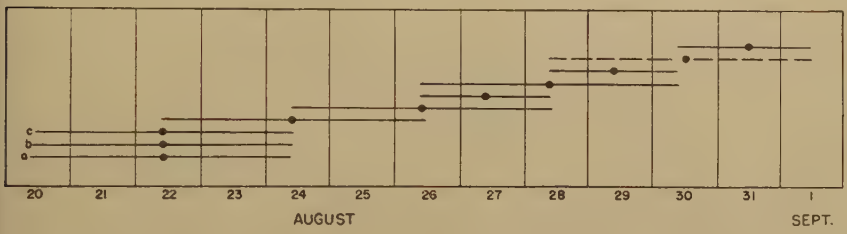


Figure 56. Plan of Experiment 1 in tank from August 20 to September 1, 1946. The horizontal lines represent the duration of exposure of bottles in determination of rates of photosynthesis. In subsequent figures, data are plotted at the midpoints of the periods. Three determinations were started on August 20 as described in text. The determination of photosynthesis during the period August 28–September 1 was unsatisfactory.

phate, temperature and pH. Salinity was measured on the first day and on several subsequent days. On some days measurements were made again in the afternoon; this was always true of temperature and pH, frequently of phosphate. The water temperature during the experiment varied between 20° and 22° C. Every second day chlorophyll was determined, and bottles were filled for starting a new photosynthesis determination. During the first part of the period these ran for four days, but starting on August 26 additional determinations were started which ran for but two days. Thus, during the latter part of the period two experiments were run concurrently. The plan is shown schematically in Fig. 56, which also serves as key to other figures, where values for mean rates determined are plotted at the midpoint of the experimental periods.

Changes in the standing crop of living phytoplankton in the tank are shown in Figs. 57 and 58 and of phosphate and oxygen in Fig. 59. At the beginning of the experiment the principal organisms in the

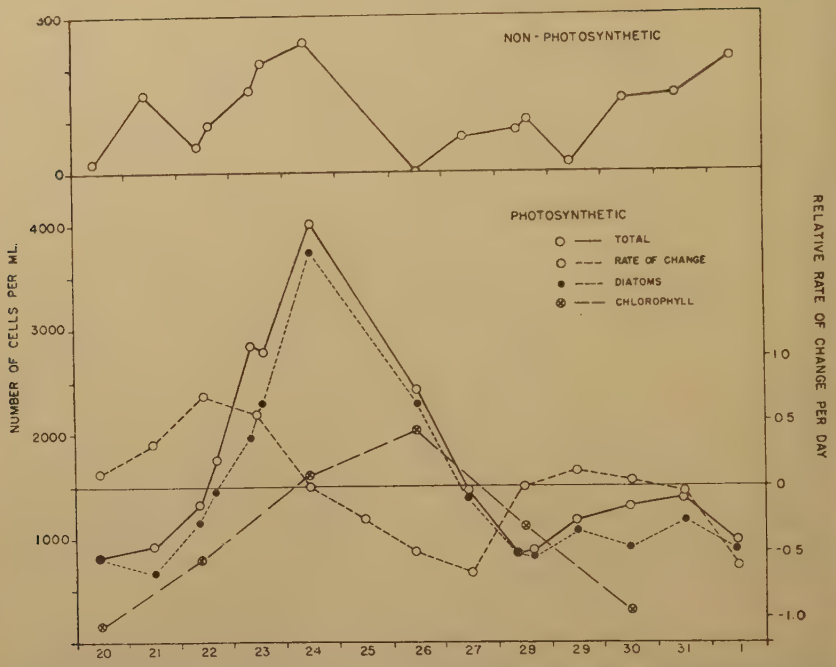


Figure 57. Numbers of phytoplankton cells (photosynthetic) and of nonphotosynthetic unicellular organisms during Experiment 1. Chlorophyll as mgm./1. × 100 is read on left scale.

phytoplankton were the diatoms *Corethron* and *Leptocylindricus*. By the end of the experimental period *Leptocylindricus* had disappeared and the diatom *Amphiprora* had become the dominant form. In addition, the dinoflagellate *Protocentrum micans* increased in numbers until August 29, after which it suffered a sharp decline. The population, as measured by number of cells, grew rapidly, reaching a peak on August 24 and then declining as rapidly. The rapidity of fall is notable but is nearly matched by some of Riley's (1943) experiments with *Nitzschia*. A smaller peak occurred on August 30. The relative rate of change of size of the population was calculated as follows: a smooth curve was drawn through the points, and from this was read the number of organisms present at the beginning of each day. The daily change, increase or decrease, was divided by the mean number present during the day. The value for August 20 was adjusted to allow for the fact that the experiment was started during the morning. The result of the calculations shows that after a period of acceleration the population was most actively growing on August 22, although the maximum

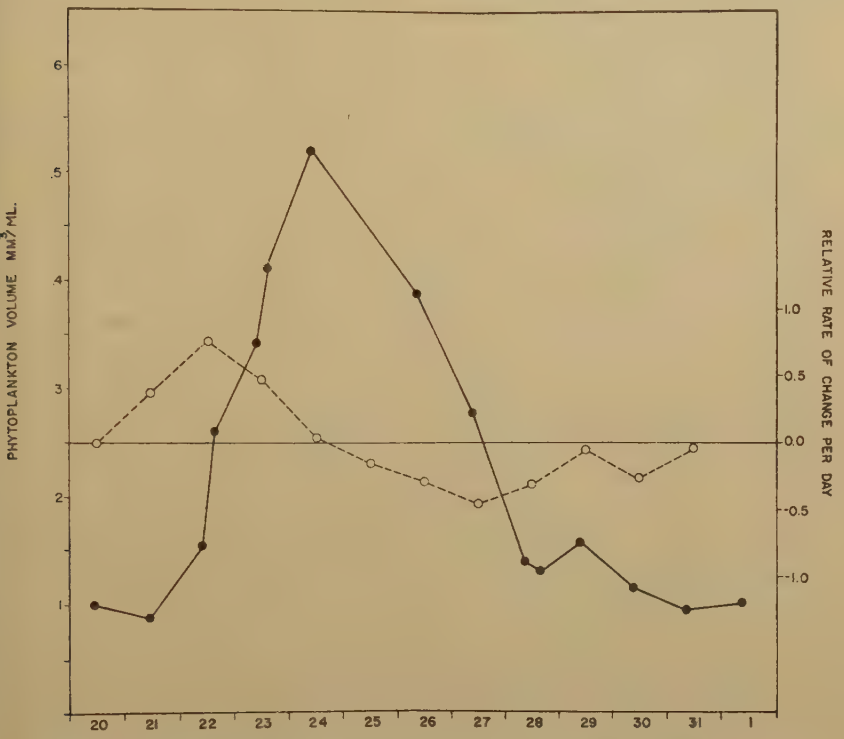


Figure 58. Phytoplankton volume during Experiment 1 as mm³ of cells per milliliter of water. Relative rate of change shown by broken line.

number was not achieved for two more days. The second peak was the result of considerably slower multiplication. The relative rate of population growth is the reproductive rate less the death rate. Since predation may be regarded as negligible here, the figures, during periods of increase, approximate the reproductive rates.

The initial stages of growth in volume of the phytoplankton were very similar, although the later stages were different (Fig. 58). The relative insignificance of the second peak, which occurred later than the peak in numbers, is accounted for by the fact that there were fewer of the largest diatoms, *Corethron* and *Rhizosolenia;* this difference reduced the volume more than the numbers of cells.

The chlorophyll content increased rather less rapidly, reaching its maximum value on August 26; there may possibly have been more on August 25 when no determination was made. The curves in Fig. 57

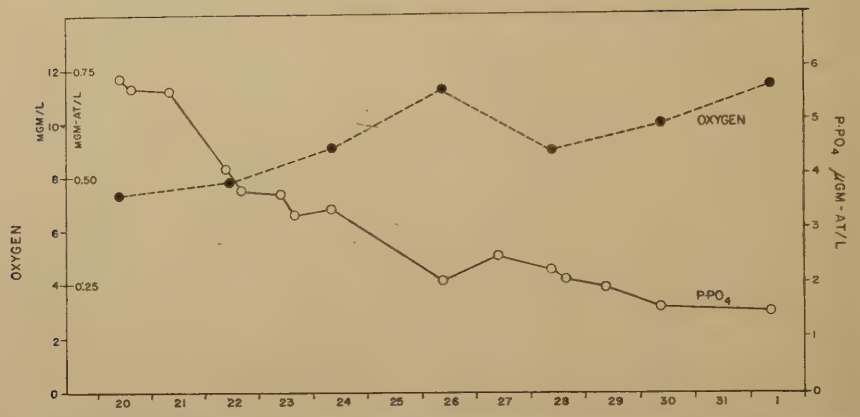


Figure 59. Concentration of phosphate-phosphorus and oxygen in the water of the tank during Experiment 1.

give the impression that the plankton was growing more rapidly than it was making chlorophyll. The second increase in plankton was not matched by chlorophyll.

The data on rate of photosynthesis are of greater interest, since they should be a measure of the rate of production of organic matter by the plankton. The immediate effect of the fertilizer is exhibited by the three measurements which were started on the first day (Table I).

TABLE I. Rate of Oxygen Changes in the Three Initial Determinations of Experiment 1 Showing the Effect of Added Nutriment. Rates of Change of Oxygen Concentration in Light and Dark Bottles and Total Rate of Production (Light—Dark) Are Given in Two Different Units. The Upper Figures Are the Rates as Milligram-Atoms per Liter per Day, the Lower Figures in Parentheses Are the Rates as Milligrams per Liter per Day

| | *Total oxygen production* | *Change in light bottle* | *Change in dark bottle* |
|---|---|---|---|
| Raw sea water (control) | 0.030 | 0.014 | −0.016 |
| | (0.48) | (0.22) | (−0.26) |
| Fertilized with $NO_3$ | 0.046 | 0.030 | −0.016 |
| | (0.74) | (0.48) | (−0.26) |
| Fertilized with $NO_3$ and $PO_4$ | 0.067 | 0.047 | −0.020 |
| | (1.07) | (0.75) | (−0.32) |

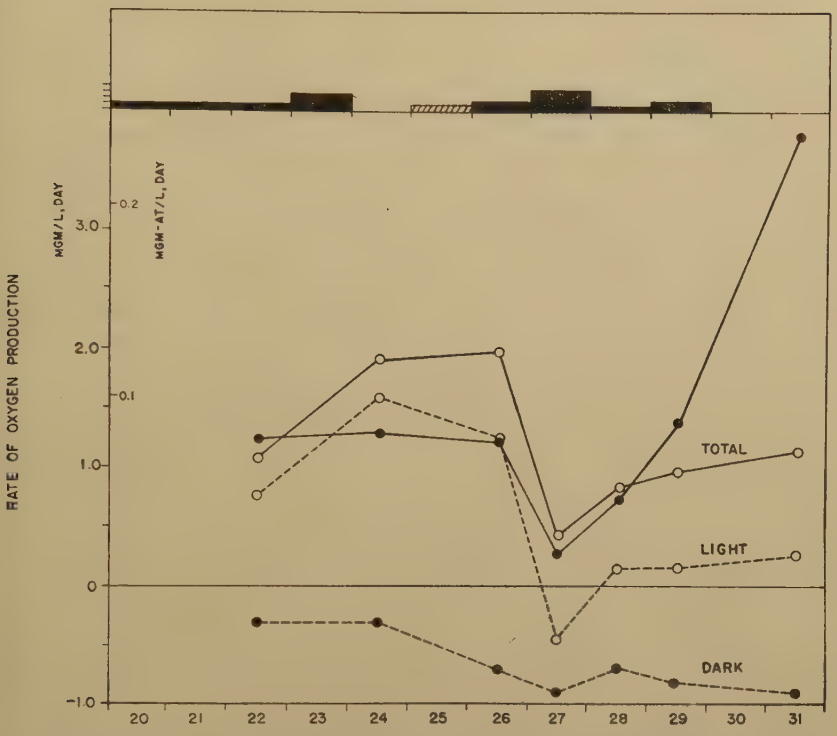


Figure 60. Rates of change of oxygen concentration in light and dark bottles and of total oxygen production (light-dark) in Experiment 1. The rate of oxygen production per milligram of chlorophyll ( $\times$ 0.01) is shown by the thick solid line and dots. Chlorophyll values used were means of concentrations in tank at beginning and end of the determination. The black band at top represents relative degree of cloudiness in qualitative units shown at left.

Addition of fertilizer had relatively little effect on respiration but increased the rate of production greatly. Addition of nitrate alone made an increase of about 50%, while further addition of phosphate more than doubled the rate in raw sea water. This seems to imply that the population before fertilization had suboptimal quantities of both nutrient substances.

The rate of photosynthesis during the following period varied greatly. The maximum rate of production occurred during the period August 22–28, which includes the date on which the population was increasing the most rapidly and on which chlorophyll concentration was greatest. The minimum rate of production occurred during the period August 26–28, which includes the day during which the

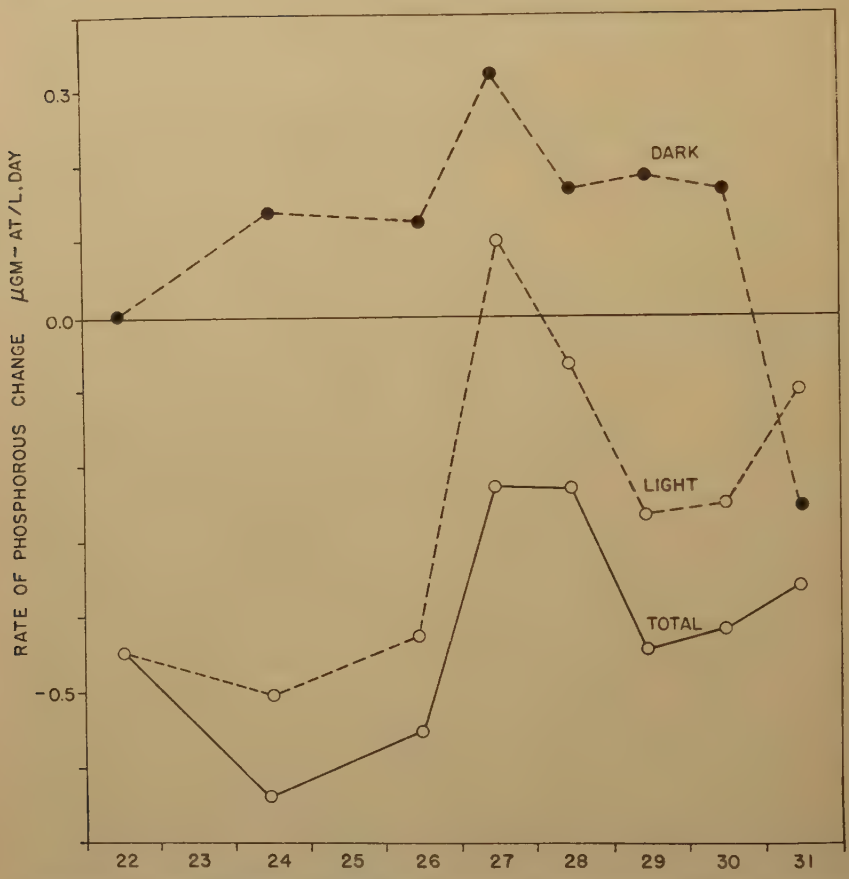


Figure 61. Rate of change of phosphate-phosphorus concentration in light and dark bottles and total consumption (light-dark).

population was decreasing most rapidly. These same rates of oxygen production were recalculated on the basis of the amount of chlorophyll by dividing by the mean amount of chlorophyll in the tank plankton during the period the bottles were exposed. The results, demonstrated by a curve in Fig. 60, show that during the first half of the experiment the rate of production per milligram of chlorophyll was rather uniform. It was very low during the period August 26–28, subsequently increased to a level slightly above the original values, and was very high during August 30–September 1. These large fluctuations are evidently due to variation in the amount of sunlight.

Unfortunately there are no actual measurements of light, but qualitative estimates of the degree of cloudiness were obtained.[5] These estimates are symbolized by the black band at the top of Fig. 60, where thicker bands correspond to more clouds and less light. Cloud and heavy rain during August 27, with moderate clouds before and after, are no doubt responsible for the very low rate of production during August 26–28, while very bright sunshine during August 30–September 1 greatly increased the rate of production per milligram of chlorophyll, although the absolute rate, per liter of water, was not so greatly increased, since there was less chlorophyll. The changes of phosphate and oxygen in the tank (Fig. 59) reflect the changes in production.

In the dark bottles the amount of phosphate always increased. In the light bottles the phosphate decreased, evidently being taken up in the processes leading to formation of new organic material. These changes were calculated as rates and handled exactly as were the oxygen data. This procedure involves assumptions not necessary when considering oxygen. Nevertheless, as seen by comparing Fig. 61 with Fig. 60, there is a very close relation between the utilization of phosphorus and the liberation of oxygen. This relationship is very clearly shown in Fig. 62, where the rates of oxygen production are plotted against the corresponding rates of phosphorus uptake. A line, fitted by eye to pass through the origin, shows that one atom of phosphorus was required under these conditions in the processes which resulted in liberation of about 90 molecules of oxygen, and, of course, fixation of that number of carbon dioxide molecules in carbohydrate. The correlation coefficient was not calculated for these few points, but it obviously would be high.

These figures give information about the ratio in which elements were used by the phytoplankton. The ratio of carbon to phosphorus contained in marine plankton may be taken as 106 : 1 by number of atoms as an average. Assuming that two atoms of oxygen are released for every carbon fixed, this ratio is 1.15 times that for the utilization by tank plankton. However, since only the oxygen that appears in the light bottle represents carbon permanently fixed in the plankton, it may be more reasonable to use the rates of increase of oxygen in the light bottle in the comparison. If this is done, it is found that the ratio in marine plankton is about 1.43 that of utilization. This is quite reasonable considering the nature of the compared figures, and the fact that the tank plankton had ample opportunity to restore any

[5] Kindly furnished by Mr. Norman T. Allen of the Woods Hole Oceanographic Institution.

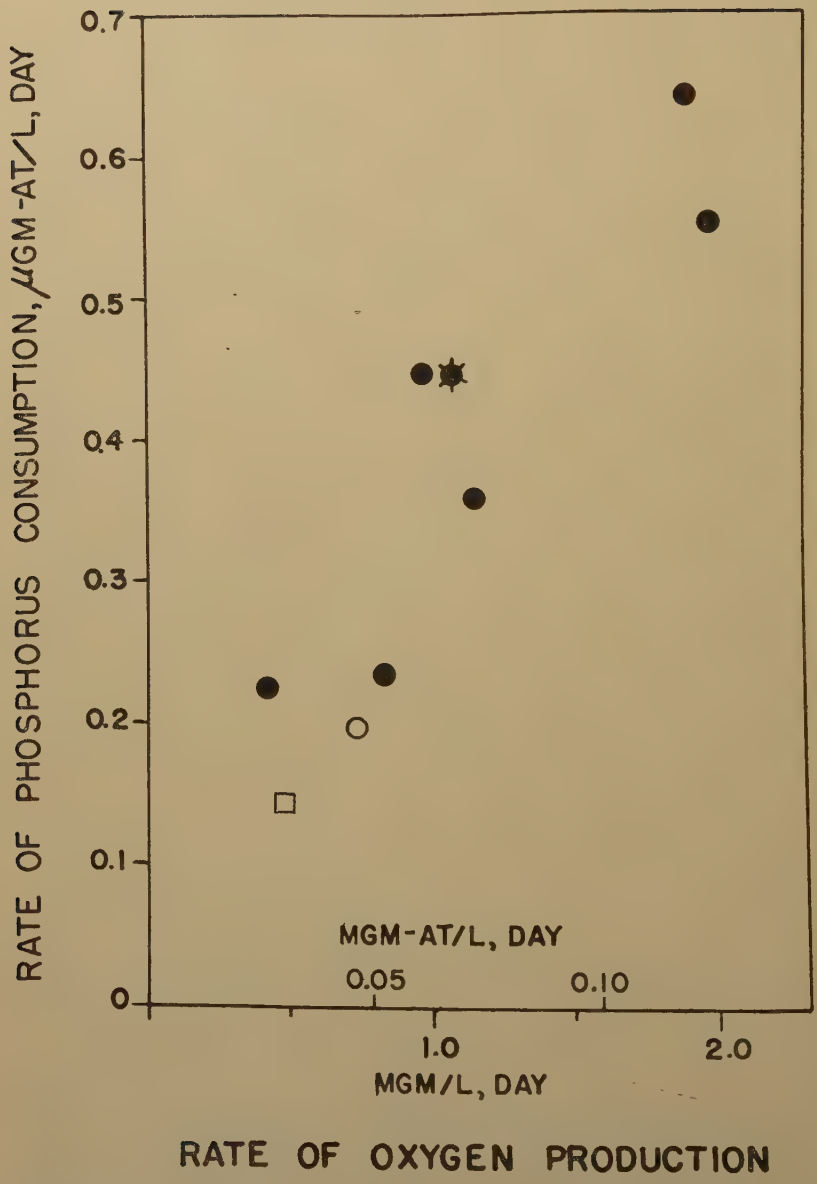


Figure 62. Correlation between total oxygen production and phosphate-phosphorus consumption in Experiment 1. Square symbol represents rates in raw sea water, open circle rates after nitrate fertilization, and starred dot after nitrate and phosphate fertilization. Other points refer to determinations on subsequent days.

phosphorus deficiencies during the initial stages of the experiment. Since data are not available on the utilization of phosphorus within the cells it is not possible to make a more exact calculation.

It was necessary to terminate the work on oxygen production by September 1, although certain observations were continued until November 29, when the tank was emptied. Twice a week temperature and phosphate were measured and plankton counted. The number of cells continued to increase, with large fluctuations, until November 7, when there were 19,000 cells/ml., whereupon the number fell to 1,800 cells/ml. on November 15. The population recovered somewhat by the time of the last collection. Despite the greater number of cells present during September, October and November as compared to August, the volume of phytoplankton remained small, fluctuating irregularly about a level of 0.03 $mm^3$/ml. This discrepancy is explained by a change in the flora which took place during September, when all the large diatoms and flagellates disappeared, leaving only small species of *Navicula, Nitzschia* and *Prorocentrum.*

This experiment forms the chief basis for the following discussion, but three others should be mentioned as furnishing additional information on the immediate effect of fertilization. The results are given in Table II.

TABLE II. Rate of Oxygen Changes in Experiments 2, 3, and 4, Showing the Effect of Added Nutriment. Rates of Change of Oxygen Concentration in Light and Dark Bottles and Total Rate of Production (Light−Dark) Are Given as Milligram-Atoms per Liter per Day. Further Explanation Is Given in Text

| *Experiment number* | *Location* | *Depth (meters)* | *Before or after fertilization* | *Oxygen Production* | | |
|---|---|---|---|---|---|---|
| | | | | *Total production* | *Change in Light* | *Change in Dark* |
| 2 | Tank | 0.5 | B | 0.031 | 0.013 | −0.018 |
| | | | A | 0.161 | 0.141 | −0.020 |
| 3 | Rand's | 1.5 | B | 0.026 | 0.005 | −0.021 |
| | Harbor | | A | 0.064 | 0.044 | −0.020 |
| 4 | Rand's | 1.5 | B | 0.010 | −0.001 | −0.011 |
| | Harbor | | A | 0.031 | 0.015 | −0.016 |

## Experiment 2

The tank was filled on July 10 in the manner previously described, and on July 11 the water was fertilized with an amount of commercial superphosphate calculated to increase the concentration by 5 μg.-at./l.

The highest concentration actually observed was 4.46. By July 15 the concentration of phosphorus was 2.62 μg.-at./l. On this day 92.7 g. of C. P. $NH_4NO_3$ were added, theoretically increasing the concentration of nitrogen by 154 μg.-at./l. Photosynthesis determinations were started just before and just after nitrate fertilization, and allowed to run four days. During this period the phytoplankton in the tank, mostly *Leptocylindricus*, *Nitzschia* and *Skeletonema* increased from 187 to 3,046 cells/ml.

### Experiment 3.

On July 25 the smaller arm of Rand's Harbor was fertilized with commercial nitrate and phosphate fertilizers in amounts calculated (assuming no tidal mixing) to increase the phosphorus by 5 μg.-at/l. and the nitrogen in the same ratio as in Experiment 1. This assumption was not seriously entertained, but it served as a basis for calculation. Just before and after fertilization, bottles for photosynthesis determinations were filled with water from 1.5 m. and suspended at that depth. The initial concentration of phosphorus was 0.87 μg.-at./l. before fertilization, 5.56 afterwards. The initial chlorophyll concentration was 0.011 mg./l.

### Experiment 4

Another experiment at Rand's Harbor was started on August 1. The smaller branch was fertilized with commercial superphosphate in an amount calculated to increase the phosphorus concentration by 25.0 mg.-at./l. Because of tidal mixing and distribution, only 4.95 was observed. The initial chlorophyll concentration was 0.010 mg./l. Photosynthesis determinations were made as in Experiment 3, and allowed to run until August 9. This long period was undesirable, but unavoidable, and the figures are probably not very representative of actual production.

## DISCUSSION

While the main object of the experiments was to investigate the extent to which fertilization may increase photosynthetic activity, this type of work should also elucidate the relationship between photosynthetic activity and changes in the standing crop of phytoplankton. Obviously, one experiment is not sufficient to furnish data for an extended discussion, particularly as major changes in crop took place over a period of time that was short as compared to the duration of the photosynthesis determinations. It may be pointed out, however, that even under these conditions a fairly consistent relation held during the latter part of the experiment. The change in standing crop was calculated as the mean of the daily relative changes in

number during the period covered by each photosynthesis determination. Not figured here, the plotted data may be described as lying quite close to a straight line, except during the first part of the experiment, when the population was presumably rapidly restoring deficiencies, growing, and when phosphorus in the water was still at a high concentration. The significance is difficult to assess with the available data, and it is hoped that this point may be attacked in further work (cf. Riley, 1943).

The pertinence of work of the sort described here to general practices of fertilization needs comment. The authors undertook this work as a study of general problems of productivity in aquatic environments rather than as a study of fertilization. Nevertheless, it is obvious that those who fertilize ponds and saline estuaries to increase fish yields must believe in the reality of nutrient chains, and in order to have a rational basis for fertilization they need information about the effect of fertilization on all steps of production, including the utilization of the added nutrients and the effect on the rate at which organic material is synthesized. It seems improbable that the effect of fertilization in actual fish ponds or natural habitats can be properly judged only by measuring the amount by which the crop of phytoplankton is increased, for the following reasons. If there is a huge increase of long duration in crop after fertilization, the implication is that phytoplankton is not being efficiently consumed, and that therefore inorganic fertilization was not required in the first place, or was insufficient alone. On the other hand, if the phytoplankton crop does not increase much, one is not sure whether the plants for some reason did not respond to the added nutrients, or whether they were kept grazed down by animals. Measurements of photosynthesis in conjunction with measurements of the standing crop would seem to yield information on this point. Measures of the number of intermediate consuming organisms are not sufficient alone to indicate browsing activity, since the mean rate of feeding per animal can vary widely during the year. To be sure, it has been well-demonstrated that it is possible to obtain satisfying yields of fish by fertilizing without studying intermediate steps, but in a scientific investigation of problems of food chains and nutrient cycles, rules of thumb devised empirically for practical purposes are not sufficient.

The determination of chlorophyll seems to the authors to be of critical importance in present day investigations of phytoplankton production. The use of chlorophyll or other plant pigments as a measure of the crop of phytoplankton has been frequently discussed (Riley, 1938; Graham, 1943; Juday, Blair and Wilda, 1943). The basis for this use of such measurements is that chlorophyll is a pig-

ment characteristic of plants, not found in animals except in food or symbiotic algae, and that it is decomposed after the death of the cell. Unfortunately, the amount of chlorophyll per cell evidently can vary, and presumably the process of decomposition is not instantaneous; therefore, chlorophyll may occur to some extent in dead cells or even outside of cells. There appears to be no satisfactory evaluation of the degree to which "dead" chlorophyll can accumulate, nor of the variations in amount per cell. In view of the known sources of error in using cell number, cell volume, and dry weight of material as indices of amount of phytoplankton, one may wonder if chlorophyll may not yet turn out to be useful at least as a supplementary measure of phytoplankton population in that it is subject to less error than some other methods.

Aside from all these considerations the more important significance of chlorophyll is that it is the gate through which practically all energy enters the phytoplankton, and hence the entire biotic association, unless there is fertilization with organic matter. Without chlorophyll, no amount of inorganic fertilizer would accomplish anything. It is therefore a measure of the synthetic potential of a plankton population. However, chlorophyll has certain limitations even as an index of photosynthetic potential, at least in comparing different localities. Attention has been directed at this point by Manning and his collaborators (Manning, Juday and Wolf, 1938; Manning and Juday, 1941), who discuss a number of aspects of the problem. The work of the Wisconsin group has been further summarized by Juday, Blair and Wilda (1943). These authors point out that the rate of photosynthesis is a rather complicated function of light intensity, varying with different species of algae and the density of population. Further, differential absorption of colors by water adds complications. Nevertheless, a better index has not yet been discovered. The expression of rate of photosynthesis per unit of chlorophyll as in Fig. 60 should permit an evaluation of the effect of light intensity upon whatever particular phytoplankton assemblage is under study, with the existing conditions of nutrition. In the papers cited above there is little mention of the nutritive conditions under which the experiments were conducted, and it is not known if the rates of production were inhibited by lack of nutrients; judging by general knowledge of nutritional conditions in nature, it may be supposed that the rates were not as high as possible under the given light and chlorophyll conditions.

It is to be expected that accumulation of information on the matters discussed here will help clarify our understanding of the effectiveness of various types of fertilizations. It may even become possible to

make diagnostic experiments with natural phytoplankton populations to determine fertilization programs in practical work.

## SUMMARY

1. The immediate effect of fertilization with phosphate or nitrate and phosphate on marine phytoplankton was studied by determining the rate of photosynthesis of the plankton before and after the addition of fertilizer. Two experiments were made in a large concrete tank and two in a salt water bay. The rate of oxygen production was increased by factors of about 1.5 to 5, although respiration was usually affected much less.

2. One of the tank experiments was maintained for a period of thirteen days, during which determinations of photosynthesis and concentration of chlorophyll were made every second day. Daily determinations of phosphate, temperature and phytoplankton concentration were made. During the initial period of rapid growth, the population increased more rapidly than did its chlorophyll. Rate of photosynthesis and growth of the plankton varied with changes in sunlight falling on the tank.

3. There was a close correlation between rate of utilization of phosphate and production of oxygen, and the ratio of the two is close to that expected on the basis of the average chemical composition of plankton organisms.

4. The use of a photoelectric colorimeter in determining chlorophyll is discussed and data presented on the transmission spectrum of the light filters.

5. The application of such studies is discussed.

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# THE POND FISHERIES OF THE PHILIPPINES[1]

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Of the various fishes commonly grown or encouraged to grow in ponds and lakes, the milkfish, or "bañgos" as it is known in the Philippines, is distinctive for its very low trophic level; in confinement it feeds mainly on the blue-green or filamentous green algae of the bottom and to a lesser extent on the submerged portions of hydrophytic spermatophytes. The pond practices in the vicinity of Malabon, where the industry is most highly developed, are designed primarily: 1) to encourage differential growth of the blue-green or green algae upon which the fish feed, at the same time maintaining aquatic conditions suitable for the fish, and 2) to exclude as nearly completely as possible all predators of this species. Nonpredacious, mildly competitive species or those of dissimilar food habits, like mullet and shrimp, are sometimes permitted and even cultured in the ponds. Any increase in production of the nonplanktonic algae can be expected to have an immediate effect in increasing the production of fish proteins, without the energy losses attendant on longer food chains, as in the case of the carnivorous species of fish. Large annual yields of milkfish are obtained in the Manila Bay region by application of the rule-of-thumb principles developed by trial and error chiefly during the past 75 years, but the basis for these various procedures has not been adequately investigated and as a result is imperfectly understood.

During the course of a survey of the Philippine fisheries made by the U. S. Fish and Wildlife Service in 1946, in which the author participated, various phases of the milkfish industry were observed. These firsthand contacts, plus the meager accounts of milkfish culture in the literature, constitute the basis for the following report.

## GENERAL

The milkfish, *Chanos chanos* (Forskål), is the only living member of the family Chanidae (Fig. 63). It is a herring-like, isospondylous fish, with a highly arched back, a single soft dorsal fin, and a large tail

[1] One of a series of papers presented at a symposium, "Fertilization of Aquatic Areas," sponsored by the Limnological Society of America at Boston, 1946.

cleft to the base. It ranges throughout the warm portions of the Indian and Pacific oceans, from southern Japan to Australia, and from southern California and Mexico to the Red Sea. An important food fish over most of its range, it is grown extensively in ponds in the Hawaiian Islands, Netherland East Indies, and the Malay Peninsula, in addition to the Philippines.

Culture of the milkfish has been carried on for more than 500 years in Java, where laws codified in 1400 A. D. provided punishment for anyone who stole these fish from a "tambak" or salt water pond. Likewise in the Hawaiian Islands the use of fish ponds goes back at least 300 years to the time of construction, according to legend, of a large fish pond at Pearl Harbor. The ponds in these localities, and in Batavia, Singapore, Madura, Formosa, and the southern Philippines, are scarcely more than fish traps, with gates or funnels permitting the fish to enter the ponds but not to return to the open sea (Fig. 64). Relatively recently the practice has developed of catching young milkfish (and sometimes mullet) along the coasts and placing them in the salt water ponds, where it is hoped they will grow to a larger size. No other care is given them. The occurrence of many predators tends to keep production low.

The specialized culture of milkfish in the central Philippines, however, employs different ponds for fry, fingerlings, and larger fish. Separate, yet closely interrelated, occupations have developed of catching the fry, raising the fry to fingerling stage, and rearing the latter fish to market size. As a result of this emphasis on the species, the milkfish ranks first in annual production in the Philippines. It is well known that fish and rice are the staples of the Filipino diet, and on good authority it is recognized that thousands of people, especially in the Manila region, depend on this one species of fish for their daily source of animal protein. During protracted periods of inclement weather in the southwest monsoon, when costal fisheries cannot be operated, the milkfish may be the only species on sale in the markets.

## BIOLOGY

Very little appears to be known about the biology of the free-living milkfish. The adults, which reach a length of 1.5 meters, live offshore and are seldom caught except during the spawning season when they are taken in small numbers by gear operated near shore. Smaller individuals up to 0.5 meter in length enter estuaries and bays with flooding tides. A small percentage make their way into some of the fresh water lakes, where they may remain for several years and grow to a considerable size. Spawning, however, occurs only in salt water offshore along sandy coasts.

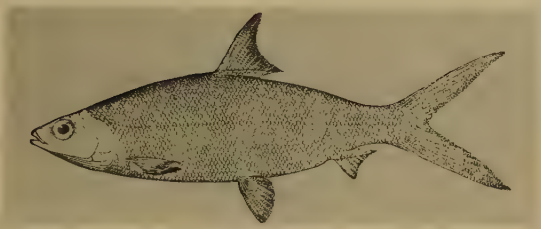

Figure 63. The milkfish or bangos, *Chanos chanos* (Forskal) (From Jordan and Evermann, 1905).

Figure 64. Trap type fish ponds on north coast of Oahu, Hawaiian Islands, showing large size of some of these ponds.

Figure 65. Seining milkfish fry from shallow water with a net of finely woven abaca fibers (sinamay cloth).

Figure 66. Funnel-shaped trap, designed to catch milkfish fry coming inshore with the flooding spring tides.

Figure 67. Larger funnel-shaped trap for catching milkfish fry. The trap is placed at the inner end of a channel connecting the sea with a shallow-water lagoon.

Figure 68. Unglazed earthenware jars (palayok) in which the milkfish fry are stored and shipped to market.

The females are said to average three to five million eggs, which are thought to be of the demersal type. Milkfish fry appear periodically along the coasts during flooding spring tides in the months March through June, but earlier in the southern Philippines than in the northern. Knowledge of the actual spawning of the adults, or of the habits and biology of the newly hatched fry before they appear along the coasts, is virtually lacking. The occurrence of these small fish only during spring tides indicates that the periodic spawning of the adults is regulated by, or at least associated with, changes in the water resulting from the monthly tidal cycle.

Milkfish fry subsist on the food reserves of the yolk sac until they reach a length of approximately 1.5 cm. The food they begin eating at this time in the ponds consists of blue-green algae and associated organisms growing on the bottom. At a length of 5 to 10 cm. the food preference begins to shift to the filamentous green algae, and proper steps must then be taken to satisfy these new dietary requirements of the fish.

## CATCHING AND HANDLING THE MILKFISH FRY

Inasmuch as the coastal fisheries represent the sole source of milkfish fry for stocking the ponds, the entire pond fish industry depends on the relatively few days each year when the fry can be caught. At such times there is a feverish activity in suitable localities, with the women and children, and to a lesser extent the men, attempting to catch the small fish by various laborious and inefficient methods (Figs. 65, 66, 67).

The newly caught fry are 10 to 15 mm. long, transparent except for the eyes, and are usually mixed in with the young of other species (mullet, tarpon, barracuda, threadfin, slipmouth, *Caranx*, etc.), which they resemble closely and from which they must be separated. Where the numbers are small this is done by picking up the fish a few at a time in a clean clam shell and discarding the undesirables, or if there are many fish to be sorted, by pouring the fish into a metal sieve with meshes 1.5 mm. square, through which only the milkfish fry can pass. The unwanted fish are usually dumped on the ground, where they die and are lost to production.

The bañgos fry, after being laboriously counted by hand in many instances, are stored in unglazed earthenware jars (palayok) of 15 to 30 liters capacity (Fig. 68) at the rate of 1,500 to 3,500 per jar, depending on its size. In this condition the fish are delivered to the market and eventually to the fishponds themselves. Usually the water in the jars is gradually freshened. The fry are said to live up to 10 or 12

days in these containers. Mortality, which ranges from 5 to 20 per cent, varies with overcrowding, protozooan parasites, length of storage, temperature, etc. No accurate records are available, but it has been estimated by members of the Philippine Bureau of Fisheries that approximately 100,000 pots of fry, representing one-third of a billion fish, are caught and marketed annually.

## POND PRACTICES

Before the recent war there were 60,000 hectares of milkfish ponds in the Philippines, all located along the coast, with the greatest concentration in the provinces bordering Manila Bay to the north of Manila. The ponds in this region are all shallow-water salt or brackish ponds. They are so constructed with reference to the mean low water level that they can be flooded with estuarine water during the incoming tide or completely drained at low tide by appropriate regulation of the head gates. Each operator usually has one or more large ponds for growing market fish, and one or more small ponds for growing fry or fingerlings or for keeping fingerling stock on hand, although some operators buy their small fish from culturists who do nothing except raise fry to the fingerling stage.

In preparation for receiving the fry, the ponds concerned are washed during several changes of tide to get rid of old salt accumulations, then completely drained and allowed to dry in the sun for several days up to two weeks, during which time the bottom is cleaned of debris and deposits thought to be undesirable. Brackish or salt water is admitted to a depth of 3 to 5 cm., and within three or four days there begins to form on the bottom a dense mat of algae and associated organisms, collectively called "lab-lab," which is the food of the bangos fry. It is said to consist primarily of unicellular and filamentous blue-green algae, chiefly of the family Oscillatoriaceae, and secondarily of a great variety of diatoms, bacteria, protozoa, and entomostraca, and a small proportion of filamentous Chlorophyceae. When this growth is well started the depth of the water is gradually increased to approximately 12 cm., and the fry are added at the rate of 500,000 per hectare in the case of productive ponds. At any time the food supply becomes low, the small fish may be transferred to another pond similarly prepared. After two to three months, when the fish have reached a length of 5 to 10 cm. and have begun to change their food habits, they are transferred to larger ponds with different food conditions.

In contrast to the fry which live entirely on blue-green algae and the associated microzoa, the fingerlings and larger milkfish feed mainly on filamentous green algae and to a lesser extent on the submerged leaves

of higher aquatics. Before the fingerlings are stocked in them, the large ponds are washed and sun dried as previously described, then flooded with salt water to a depth of 10 to 15 cm. With this greater initial depth of water the lab-lab complex does not develop, but there appears instead a heavy growth of filamentous green algae, known as "lumut," consisting mainly of *Cladophora, Enteromorpha, Vaucheria, Spirogyra, Chaetomorpha,* and *Oedogonium* (Herre and Mendoza, 1929). After the lumut in the growing ponds is well started the depth of water is gradually increased to 30 to 50 cm. Usually a trench about one meter deep is provided in each pond to afford protection for the fish from the hot midday sun, but this is about the greatest depth of water permitted in the ponds.

Although lumut is the main food of the growing milkfish, rooted aquatics are also eaten when they are present. These plants, known collectively as "digman," comprise *Ruppia graminea* and *Halophila beccarii,* which are said to prefer salt water, and *Nejas graminea, Halophila ovata,* and *Thalassia hemprichii,* which are said to grow best during the rainy season when the salinity of the ponds is decreased. The roots of *Lemna paucicostata* are also eaten by the fish, but since this plant grows in water which is almost fresh, it is not of wide occurrence in the milkfish ponds where the salinity is kept fairly high.

## PRODUCTION OF FISH IN THE PONDS

Under favorable conditions the milkfish grow rapidly in the ponds, reaching a length of 25 cm. in four months after the fingerlings are stocked in the growing ponds, 40 cm. in eight months, and 50 cm. in a year (Seale, 1908). However, because the greatest demand is from the Filipinos of the lowest income group who cannot afford to buy big fish, the bañgos are usually marketed after they have reached a weight of 300 to 600 grams. Six to nine months are sufficient to grow fish of this size.

Either one or two crops per year are produced in the ponds. In the former instance the fish are marketed at the end of the calendar year when they have reached a weight of 400 to 600 grams, whereas in the latter instance they are marketed in September and October and again in April and May after an individual weight of 300 to 500 grams has been attained. Accepting the figures of Adams, Montalban and Martin (1932) that 900 to 1000 market fish can be raised per hectare of pond surface, the average annual yield of a productive series of ponds is thus 500 to 1000 kg. per ha., depending in part upon the system of cropping employed.

Average mortality from all causes is reported to be approximately

70 per cent in the central Luzon fish ponds (Adams, Montalban and Martin, 1932), and proportionately greater in the ponds of the southern Philippines where less care is taken of the growing fish and where predacious species are frequently abundant in the ponds. Assuming an annual yield of 750 kg. per ha., which is undoubtedly generous, the annual production of this species, excluding the relatively small proportion of market size fish caught in the coastal fisheries, would be 45,000,000 kg.; and assuming each fish weighed 500 g. at the time of marketing, the number of fish marketed—90 million—would be hardly one quarter of the estimated number of fry caught each year. Actually the 1938 census listed an annual production of only 14,530,000 kg. of milkfish.

## DISCUSSION

The annual production of various species of fish depends mainly upon the trophic levels of the species (Lindeman, 1942), other conditions being equal. The primary consumers (those feeding on autotrophic plants) have a greater annual production than the secondary consumers (those feeding mainly on primary consumers), and they in turn have a greater annual production than tertiary consumers, etc. Swingle and Smith (1941) found, for example, that the production of goldfish and shiners (primary consumers) varied from 750 to 1,100 pounds per acre, of bluegills and bullheads (secondary consumers) from 500 to 600 pounds per acre, and of largemouth bass and white crappies (tertiary consumers) from 150 to 200 pounds per acre. Thompson (1941) observed approximately the same relationships among standing crops of fish in 22 Illinois ponds and lakes, noting in addition that in general five pounds of food were required to produce a pound of fish.

Because the milkfish is at the third trophic level in the energy cycle, feeding directly on the primary producers (algae and higher aquatics), it would be expected *a priori* to have a high productivity; and when this fish is raised in ponds where the food plants are abundant and enemies largely excluded, such an expectation is realized. The annual production of 500 to 1,000 kg. per ha. in the Philippines is a substantial yield, considering that it has resulted entirely from food grown in the ponds without the addition of any fertilizer. It is greater by far than the annual production of sport and commercial species in any of the lakes and ponds considered by Rounsefell (1946) and considerably greater than the carrying capacities of nearly all these bodies of water. It is greater than the average annual yields obtained by Swingle and Smith (1941) in fertilized ponds, although in this instance production would be expected to be lower because the fish concerned are from a

higher trophic level than the milkfish. It is considerably less, however, than the annual yield of fresh-water ponds in southern China, where carp and other species are fattened on foodstuffs added to the ponds. Annual yields up to two or more tons per acre are reported in this fishery.

With the milkfish directly dependent on plants for its energy, any increase in the growth of these food organisms would logically result in an immediate increase in production of fish within the limits of the particular ponds. Fertilization would appear to be the next step in the Filipino system of milkfish culture, but here the problem is not the same as in other pond cultures where fertilizers have been used effectively to increase fish production. The goal in the present instance is not to increase the quantity of phytoplankton, zooplankton, or bottom organisms, but rather the amount of matted Myxophyceae and filamentous Chlorophyceae growing on the bottom. Our present knowledge of the nutritional and environmental requirements of the various groups of algae does not enable us in most instances to selectively culture in ponds the particular types desired.

In the Philippine pond system both qualitative and quantitative growth of the algae appears to result from the interaction of at least three factors: nature of the pond bottom, depth of water, and salinity, with the first mentioned factor probably the essential basis of present productivity.

Rosell and Argüelles (1936) concluded from an analysis of 33 bottom samples from milkfish ponds in the Manila Bay region that "algae seem to grow very abundantly in types of subaqueous horizons that have a high solution loss and a high content of clay, nitrogen, and organic matter. Types that are deficient in these characteristics generally have very few algae." Greatest production of algae was noted in ponds having bottoms described as "peaty clay, either compact or matted in structure" or "Slimy clay, slightly organic and gelatinous in consistency." Least production was associated with bottoms of "Sandy mud, sand-shell admixed" and "Sand, clean sand compact." This is another variation on an old theme long known to the European carp culturists (Schäperclaus, 1933), that the pond bottom is at least equal in importance to the overlying water in determining production of fish. Basic productivity varies from one region to another, depending not only on the geological origin and subsequent history of the surface deposits (regional limnology) but also on the colloid complex of the pond bottom which is so important in base exchange and retention of essential nutrients.

Variation between ponds in production of algae has been a source of considerable trouble to the Filipinos. Some ponds are so productive

that it is claimed the growths of algae must be thinned out to permit the fish to move about. In others, which do not share in this exuberant growth of plants, *Oedogonium* and other kinds may be imported and placed loose in the pond, or clumps of *Cladophora* and forms with similar growth habits may be planted in the bottom, but these procedures ignore the cause of the low productivity and probably satisfy the culturists more than the hungry fish. Sometimes rice bran (tikitiki) soaked in water is used to supplement the natural food of the fish in unproductive ponds.

Drying of the ponds at regular intervals is likewise practiced by the European pond culturists (Schäperclaus, 1933) for the reason that the aerobic processes which are favored by this procedure result in an oxidation of the organic matter with a consequent liberation of plant nutrients to the colloid complex ("mineralization"), and an improvement in structure of the bottom colloids. In the short periods of drying in the Philippines it is likely that the fixation of nitrogen by blue-green algae, certain bacteria in their mucilaginous sheaths, and by the *Azotobacter*-group proceeds apace. The published reports on the Philippine practices, however, merely generalize that the drying promotes the growth of lab-lab and kills many of the enemies of the bañgos, without offering any explanations as to the mechanism of the former effect.

A statement frequently occurring in the literature is that the selective growth of the blue-greens or greens depends primarily upon the initial depth of water at the time of flooding; very shallow water encourages the Myxophyceae, whereas somewhat deeper water encourages the filamentous Chlorophyceae. Adams, Montalban and Martin (1932) caution against flooding the ponds deeper than 12 cm. after the lab-lab complex has begun a good growth, otherwise the filamentous green algae which cannot be utilized by the fry will start to develop. The fact that the same pond can be used for production of lab-lab and later for the production of lumut by altering the technique of flooding is further substantiation of the point in question.

Depth of water is obviously only one of many covarying factors influencing the growth of the algae. Although the depths are very small it is possible that differences in light penetration and temperature might contribute to the differential production of greens or blue-greens. In very shallow streams with dark bottoms, for instance, water temperatures considerably higher than those of the adjacent air were recorded by the author in the Philippines. It is quite likely that conditions at the shallower depths of water are more favorable for the growth of the nitrogen-fixing Myxophyceae and their associated bacteria. Assuming that most of the plant nutrients come from the

colloid complex of the subaqueous horizon, it might be expected that the concentration of these nutrients would be greater after a given period of time in a small volume of water than in a larger (deeper) volume.

At present no fertilizers are being added to the Philippine fish ponds in an attempt to increase production of algae, and even if the fertilizers were available in adequate quantities in the islands, it is more likely they would be applied to the deficient agricultural soils than to the already productive ponds. In effect, though, there already is a type of fertilization in that the water of the fry ponds, and to a lesser extent that of the growing ponds, is replaced at frequent intervals. Where these ponds are located along streams draining population centers it is likely, although studies have not yet been made to ascertain the truth of this, that considerable quantities of nutrients are brought into the pond with each exchange of water. Gross observations on regional variation in pond productivity tend to support this view.

Although bañgos can be grown in fresh water ponds or in salt water ponds, including any part of the continuous range in salinity between these two extremes, they apparently do best in strongly brackish water because of better growth conditions there for the food plants. Roxas and Umali (1937) observed that milkfish and mullet "are unprofitable to culture in fresh water ponds, where the growth of algae, the bulk of the food of these two species, is entirely inadequate to supply their needs for rapid growth." At times, however, the water in the ponds must be fresh or nearly so to permit significant growth of such forms as *Spirogyra*, *Oedogonium*, and *Lemna*. The various components of lab-lab and lumut have different salinity tolerances, so that the composition of the association at any particular time will depend in part upon the salt content of the pond water, and this varies from pond to pond and from season to season.

## SUMMARY

By way of summary, the Filipinos have developed a successful method of rearing milkfish in ponds by rule-of-thumb methods without understanding the basis for the various procedures employed. Production can be still further increased by utilizing for fish pond construction even a small portion of the 400,000 hectares of mangrove swamps in the islands. Perhaps production can also be increased by a program of careful fertilization worked out experimentally to determine the economic as well as biological feasibility. At least in an instance such as this, where the flow of energy in the food chain proceeds directly from alga to fish, the possibilities of so increasing production are sufficiently great to merit investigation.

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# OXYGEN CONSUMPTION IN SEA WATER OVER LONG PERIODS[1]

By

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The rate and extent of oxygen consumption in sea water have been studied and measured under a variety of conditions and for a variety of purposes. Such studies, of what is often called the "biological oxygen demand," have either been involved in the estimation of the total content of organic matter or have contributed to the understanding of the processes of organic decomposition in the sea. They have usually consisted of bottling samples of the water and storing them for varying periods, generally at controlled temperatures, determining the oxygen content of the samples and comparing it with the content of the original water. The incubation time has seldom exceeded a few days, or at most a few weeks.

It has generally been supposed that oxygen consumption will continue, perhaps indefinitely, at a diminishing rate. But no one has apparently determined how long it will take for the process to become entirely imperceptible.

As would be expected, the rate and extent of oxygen consumption depend upon the temperature, the time of storage, and the source of the water, which determines the amount and character of the organic matter present.

Oxygen consumption is one of the factors concerned in the distribution of oxygen in the sea, and especially the occurrence and significance of the "oxygen minimum." Sverdrup (1938) has shown the mathematical consequences of the various possible assumptions concerning the rate of oxygen consumption and the distribution of oxidizable organic matter. Seiwell (1937) determined the oxygen consumption in sea water samples from various depths, incubated for ten days at two temperatures, 11° and 25° C. Extrapolating the time, he calculated the annual consumption at temperatures *in situ*. His results were many times in excess of values which he had calculated from the

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study of the oxygen distribution in the sea, and his conclusion was that in the ocean there are restrictive influences which limit oxygen consumption.

While it is to be expected that experimental laboratory conditions will yield results very different from those in the ocean, it seems that the closest approximation to the natural process would be to take sea water samples from various depths and to incubate them for as long a time as possible at the temperature which the water had *in situ*, removing samples for analysis periodically.

Water was obtained from two stations, "B" and "C," located respectively at 39° 05′ N., 70° 40′ W. and at 35° 30′ N., 67° 30′ W. The most significant part of the water column is the oxygen minimum layer. Samples were accordingly taken from this level (230–250 m. at Station B and 900–920 m. at Station C), as well as from the surface and from a depth well below the minimum (1240 m. at Station B and 1725 m. at Station C). The "minimum" samples were immediately placed in glass-stoppered bottles of about 250 ml. capacity and incubated in a water bath in the dark at 8° to 9° C., their original temperature. The deep samples were similarly stored at a temperature of 4° to 5° C. The surface samples were distributed, some of them at 8°, some at 4°, and some at the uncontrolled laboratory temperature, about 25° C. Bottles were removed for analysis periodically over a period of nearly two years (713 days).

All analyses were made in duplicate (from a single bottle in storage) by the routine Winkler method. A single solution of sodium thiosulfate was used for the whole period and its concentration determined before each analysis by titration against standard $KIO_3$ and $K_2Cr_2O_7$. Several standards were prepared during the experiment and cross-checked against each other.

Due to misunderstanding, different solutions were used in the original determinations of oxygen content, made on shipboard, so that the first two analyses for each sample were considered unreliable and are not included in the results given. However, since the object of the experiment was to follow the long-term changes, rather than those which took place in the first few days, this is not considered a serious gap in the data.

The results are shown in Table I. It will be seen that the surface samples, from both stations, were erratic, especially those incubated at 25°. The surface samples incubated at each of the lower temperatures were more regular and came to a constant level (depending on the temperature) after about 100 days of incubation. Significantly greater consumption resulted at the higher temperature, even after two years.

TABLE I. Consumption of Oxygen ($O_2$ ml./L) in Water from Three Different Levels at Two Stations. Water Was Stored in the Dark at Temperatures (C.) Close to Those *in situ*, in Small Individual Bottles, One of Which Was Used for Each Analysis. Time Is Measured from the Day the Water Was Sampled

| *Time* (*days*) | *Station B* | | | | | *Station C* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *Surface* | | | *"Minimum"* (*230–250 m.*) | *Deep* (*1240 m.*) | *Surface* | | | *"Minimum"* (*900–920 m.*) | *Deep* (*1725 m.*) |
| | *25°* | *8–9°* | *4–5°* | *8–9°* | *4–5°* | *25°* | *8–9°* | *4–5°* | *8–9°* | *4–5°* |
| 18 | 4.11 | 4.50 | 4.68 | 3.55 | 6.38 | 3.62 | 4.05 | 4.15 | 3.54 | 6.18 |
| 42 | 4.18 | 4.28 | 4.15 | 3.50* | 6.13 | 1.49 | 3.40 | 4.10 | 3.43* | 6.07 |
| 66 | 3.83 | 4.16 | 4.08 | 3.34 | 6.06 | 3.25 | 3.92 | 3.95 | 3.43 | 5.99 |
| 85 | 3.93 | 4.22 | 3.96 | 3.26 | 6.02 | 1.21 | 3.44 | 3.79 | 3.43 | 5.98 |
| 113 | 3.85 | 3.78 | 3.85 | 3.27 | 6.14 | 3.09 | 3.76 | 3.77 | 3.40 | 5.95 |
| 148 | — | — | — | 3.31 | 6.12 | — | — | — | 3.43 | 6.04 |
| 200 | — | 4.06 | — | 3.21 | 6.13 | — | 3.43 | — | 3.41 | 5.98 |
| 260 | 3.51 | — | 3.76 | 3.32 | 6.23 | 2.60 | 3.06 | — | 3.44 | 5.97 |
| 323 | — | — | — | 3.48 | 6.05 | — | — | — | 3.40 | 6.06 |
| 412 | — | 3.99 | 3.75 | 3.47 | 6.12 | — | 3.15 | 3.80 | 3.42 | 6.03 |
| 583 | — | 4.11 | 3.66 | — | 6.14 | 1.74 | 3.14 | — | 3.44 | 6.02 |
| 713 | — | 4.15 | 3.53 | 3.46 | 6.11 | 2.89 | — | 3.77 | 3.39 | 6.01 |

* At this point samples of this water were raised to room temperature (25° C.); results are shown in Table II.

The most striking fact observed is that within the limits of the analyses there is no net oxygen consumption in either the "minimum" or the "deep" samples after the first 50 days. They remained without change for nearly two years at their temperatures *in situ*. The deep samples were particularly regular in behavior.

The apparent rise in oxygen content in the minimum samples from Station B, after the initial drop, is very likely the result of the conditions of the experiment, and the abnormally large oxygen consumption in some of the surface samples incubated at 25° is presumably due to the accidental inclusion of plankton material or other easily oxidizable substances.

In order to determine the relative importance of the temperature, some of the "minimum" samples were removed after 42 days and incubated thereafter at 25°. The results, shown in Table II, indicate consistently greater oxygen consumption in these samples.

A second series of experiments was begun, primarily to study the

TABLE II. Consumption of Oxygen ($0_2$ ml./L) at Room Temperature (25° C.) in Water from the Oxygen Minimum Layer at Two Stations. Water Had Previously Stood for 42 Days at 8–9° C. (See Note to Table I)

| *Time (days)* | *Station B* | *Station C* |
|---|---|---|
| 0 | 3.50 | 3.43 |
| 6 | 3.34 | 3.34 |
| 20 | 3.27 | 3.33 |
| 39 | 3.13 | 3.40 |
| 69 | 3.10 | — |
| 212 | 2.96 | 3.27 |

TABLE III. Consumption of Oxygen ($0_2$ ml./L) in Water Sampled and Stored Under Conditions Similar to Those in Table I. Time Measured from the Day the Water Was Sampled

| *Time (days)* | *Surface* | *"Minimum" (400 m.)* | *Deep (1200 m.)* |
|---|---|---|---|
| | *25°* | *8–9°* | *4–5°* |
| 0 | 5.11 | 3.89 | 6.28 |
| 2 | 5.11 | 3.84 | 6.21 |
| 20 | 4.97 | 3.76 | 6.17 |
| 40 | 4.80 | 3.75 | — |
| 82 | 4.41 | 3.76 | 6.13 |
| 172 | 4.65 | 3.81 | 6.10 |
| 305 | 4.50 | — | 6.14 |

consumption in the first few days. Samples were taken from three levels, as before: surface, 400 m. (the oxygen minimum level), and 1200 m. They were incubated at the same temperatures as in the first series (which were the temperatures *in situ*). The results are shown in Table III. The surface samples again behaved erratically, with oxygen consumption continuing for about 80 days. In each of the other sets of samples, oxygen consumption ceased after the first few days.

These experiments do not, of course, give us a clear picture of the process of oxygen consumption *in situ*. One wonders whether the true, natural rate of consumption is the maximum rate noted in the first few days of laboratory incubation or the minimum rate integrated over the whole period. The former assumption leads to the anomaly obtained by Seiwell and already referred to—more oxygen apparently consumed than can conceivably be available. Rather than to attempt to reconcile the situation by assuming *retarding* agencies in the sea, it is more in keeping with known facts to assume that the laboratory

conditions *hasten* the normal rate of consumption. Zobell and Anderson (1936), and others, have found bacterial action to be stimulated in bottled samples, because of surface effects. It is impossible to tell to what extent the observed initial rate of consumption is due to such surface concentration of bacteria, but it is probably not entirely due to this.

One also wonders why oxygen consumption ceases after a comparatively short period. It is evidently not the result of complete decomposition of organic matter. Among others, Waksman and Carey (1935) have pointed out that bacteria do not attack and oxidize all the organic matter in the water. Further, they discuss the "dynamic equilibrium" in sea water, involving the organic matter and its decomposition products, the oxygen tension, the temperature, the bacterial population, etc. The unknown condition of the equilibrium must be responsible for the fact that in water which has come to a steady state at low temperature additional oxygen is consumed when the temperature is raised.

It may be significant to observe that the surface samples which were incubated at the lower temperatures eventually reached a level of oxygen concentration similar to that in the "minimum" samples. This lends some support to the view that the organic matter in a vertical water column (at least down to the oxygen minimum layer) is principally derived from the local surface.

Until we know more about the nature and distribution of the dissolved organic matter in the sea, we cannot safely translate such laboratory experiments as these into natural processes. Nevertheless, there is reason to suspect that the natural process of exygen consumption in the sea is slower than we may previously have assumed.

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# MAXIMUM AND MINIMUM MONTHLY MEAN SEA SURFACE TEMPERATURES CHARTED FROM THE "WORLD ATLAS OF SEA SURFACE TEMPERATURES"[1]

By

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## INTRODUCTION

Maximum and minimum water temperatures are of interest in oceanography in many connections. Among their more important uses, they afford the most valuable simple index of temperature conditions for analyses of the distributions of marine organisms.

The "World Atlas of Sea Surface Temperatures," issued by the Hydrographic Office (1944) of the United States Navy Department, incorporates a large share of the previously published temperature data, together with records in the Hydrographic Office files totalling in the millions. Since the Atlas is therefore the most comprehensive treatment of surface temperatures available, and is likely to be a standard reference for some time, preparation of maximum and minimum charts conforming to its data seemed desirable. Such charts are given here in Plates I and II. The method of preparing them is described below, followed by some comments on the degree of reliability which should be accorded them and the Atlas.

## METHOD OF PREPARATION

The Atlas plots the isotherms of monthly mean temperatures of even 5° F. intervals, giving monthly charts for the Atlantic, eastern Pacific, western Pacific and Indian Ocean sectors of the globe. Isotherms for intermediate 2½° F. values are also given for some months in some regions, but these have not been used here.

From the monthly Atlas sheets for an ocean, a separate map was traced for each 5° temperature value, superimposing the courses of all its isotherms during the twelve months. Figure 69 illustrates the procedure with the isotherms of 75° F. in the South Atlantic. The whole area occupied at any time during the year by a particular monthly mean temperature was thereby determined. The distribu-

[1] Contribution No. 397 from the Woods Hole Oceanographic Institution.

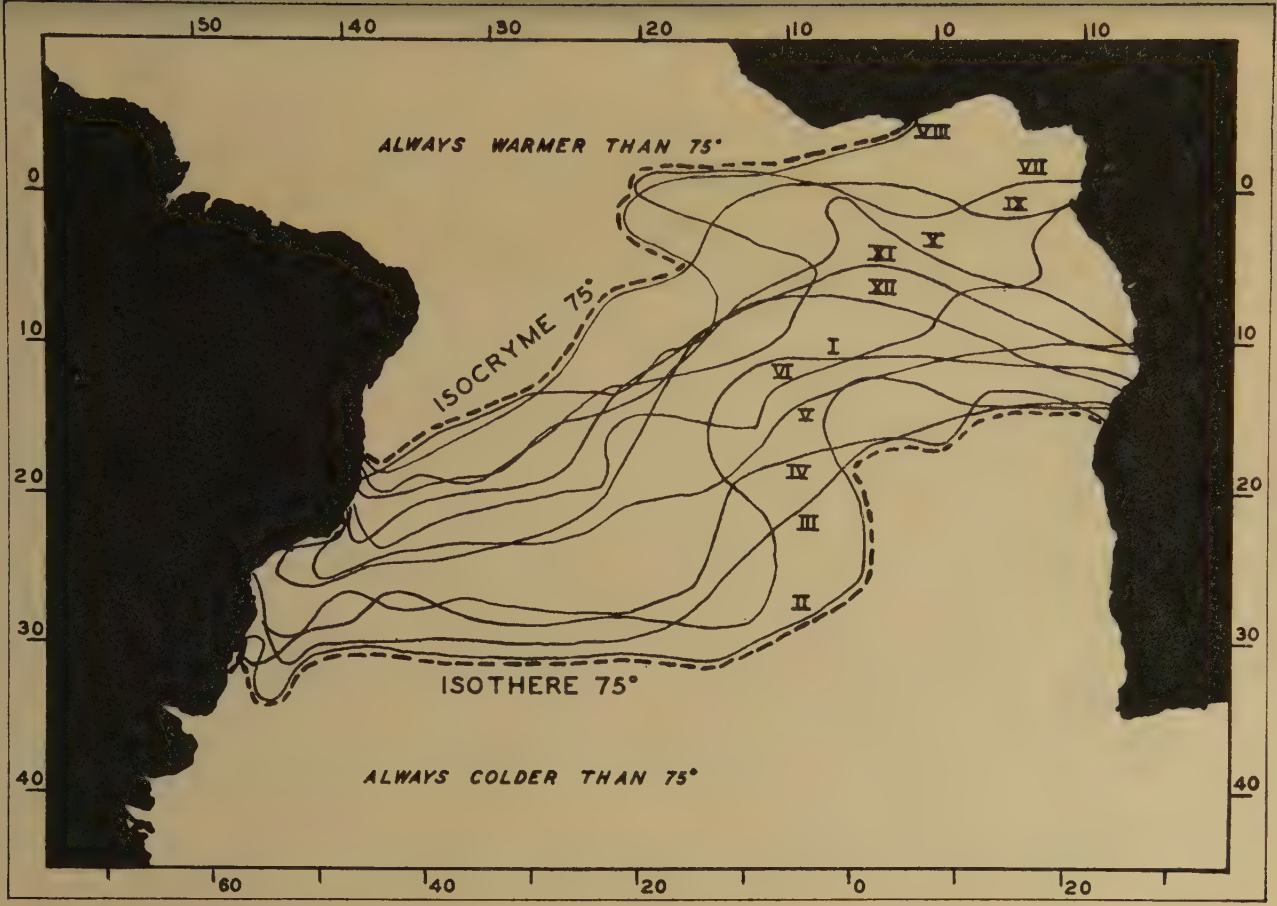


Figure 69. Illustration of the method of determining the courses of isotheres and isocrymes given in Plates I and II. The isotherms of 75° F. for each of the twelve months in the South Atlantic are superimposed on one chart. An area is thereby determined in which the surface water is sometimes more and sometimes less than 75°. The equatorward boundary of the area provides the isocryme, the isotherm of 75° at the time of greatest cooling, irrespective of the month in which this occurs. The poleward boundary of the area provides the isothere, or isotherm of maximum warming, in the same way.

tion of maximum monthly mean temperatures was determined from the poleward limits of all such areas and the distribution of minimum means from the equatorward limits.

Plates I and II thus give the maximum and minimum monthly mean temperatures irrespective of the months in which these occur. Their isotherms, in other words, connect points which warm up or cool down to the same extremes as measured in monthly mean temperatures. Such isotherms are known as ISOTHERES (equal warmth), and ISOCRYMES (equal coldness). Nominal calendar months being the basis of all calculations, rather than the warmest and coldest 30-day periods, the designations CALENDAR MONTH ISOTHERES and CALENDAR MONTH ISOCRYMES are appropriate.

It may be emphasized that Plates I and II give monthly means. Of course absolute maximum and minimum temperatures at any locality would be several degrees more extreme.

One exception to exact conformity with the Atlas has been made. Fuglister (1947) has reviewed more recently all of the data for the western North Atlantic and has prepared revised charts of that region. Plates I and II conform with his delineations of the maximum and minimum monthly means for the area (Fuglister, 1947: pl. 13, 14). On the whole, these do not differ significantly from the isotheres and isocrymes determinable from the Atlas alone.

Mercator projection has been used for Plates I and II instead of the sinusoidal equal-area projection of the Atlas. Hydrographic Office Chart 1262B served as the main base map. The arrangement whereby both the oceans and the continents are shown as unbroken masses appears to be the best suited to most general oceanographic and biogeographic purposes.

## ACCURACY OF THE WORLD ATLAS AND OF PLATES I AND II

The Atlas, and the present charts of isotheres and isocrymes, appear to be reasonably reliable in the general patterns of temperature distribution shown. The highest sustained temperatures for the year round are found in the Indo-West Pacific Tropics, and in small sectors of the eastern Pacific and the western Atlantic Tropics, all of which have monthly means always in excess of 80° F. and mostly between 80° and 85°. The maximum temperatures, on the other hand, are found in the Persian Gulf, where monthly means exceeding 90° F. occur in summer, although winter monthly means in part of this area fall to about 60°. The greatest annual range, more than 40°, is found in the inner end of the Yellow Sea, where maximum means in excess of 75° and minimum means of less than 35° occur in the Gulf of Pohai.

Major upwellings and other large features of circulation in the surface waters are indicated by pockets, inflections and deviations of the individual isotherms. Not all irregularities of the isotherms are to be regarded as equally significant in this way, however, some resulting from peculiarities in the data or from procedural techniques such as charting the isotheres and isocrymes with as little smoothing as possible. It can be doubted, for instance, that the three warm spots enclosed by the isocrymes of 80° on the west coast of Central America are in reality as complex as the chart indicates.

Commentaries on the over-all accuracy of the Atlas are given by its compilers, and, indirectly, by Fuglister (1947). The former state that the delineation of isotherms for intervals smaller than 5° F. generally would give a false impression of the adequacy of the source data. Fuglister discusses the distortions introduced by the method used in compiling and reducing data, in which random numbers of observations for various years are averaged by months for one-degree quadrangles of latitude and longitude. Steep gradients tend to be spread horizontally in this technique, partly by use of the quadrangle as the smallest measure of area, partly because the positions of gradients may vary somewhat from time to time, and partly because of inaccurate fixes of observations.

Comparison of the charts with detailed local data for a number of points in Europe, America, and Africa shows that the Atlas information is only approximately correct for the most inshore waters. This, of course, is unavoidable, partly for the foregoing reasons and partly because local detail can not be charted on a world map of convenient scale. Caution is required, therefore, in using the present charts in biological investigations involving shore collections, or in similar applications.

Inadequacies of the source data of the Atlas, chiefly in higher latitudes, probably account for certain other limitations which should be noted. Interpolated isotherms are given in some cases and contribute portions of the isotheres and isocrymes of the mean ice limit, 35°, and 40° in Antarctic waters. Elsewhere, occasionally no isotherms are provided. For such reasons it has been impossible to chart the 55° isothere in the Sea of Okhotsk, the 35° isothere north of Bering Strait, and the blank portions of the mean ice limits. The delineations in the Arctic, particularly those for summer regarding the Siberian and Canadian coasts from the Kara Sea to Baffin Bay inclusive, must be taken with considerable reservations. The U. S. Hydrographic Office (1946) "Ice Atlas of the Northern Hemisphere" does not indicate that a majority of these coastlines are ice bound in the warmest months, as the World Atlas suggests, but does plot a large, persistent free ice

field in western Baffin Bay which can not be reconciled with the course of the 35° and 40° isotheres there. These and other discrepancies are partly offset by the fact that there are great year-to-year variations around the average conditions in the Arctic, notably for ice distribution, and delineations of mean positions of isotherms there are at best of only provisional interest.

In preparing Plates I and II it has been found that a few isotherms in the Atlas do not join perfectly from one sector of the oceans to the next. The differences are seldom more than 2° of latitude, however, and are of no great consequence. Smoothed transitions across the overlaps have been used where necessary for the isotheres and isocrymes.

The collection of more extensive data will doubtless warrant eventual revision of at least portions of the thermal picture in Plates I and II. If judiciously used, nevertheless, the present charts are serviceable for most purposes to which they can be put. Over a year of testing, they have proved highly satisfactory for the analysis of biological distributions as carried out in this laboratory (Hutchins, 1947).

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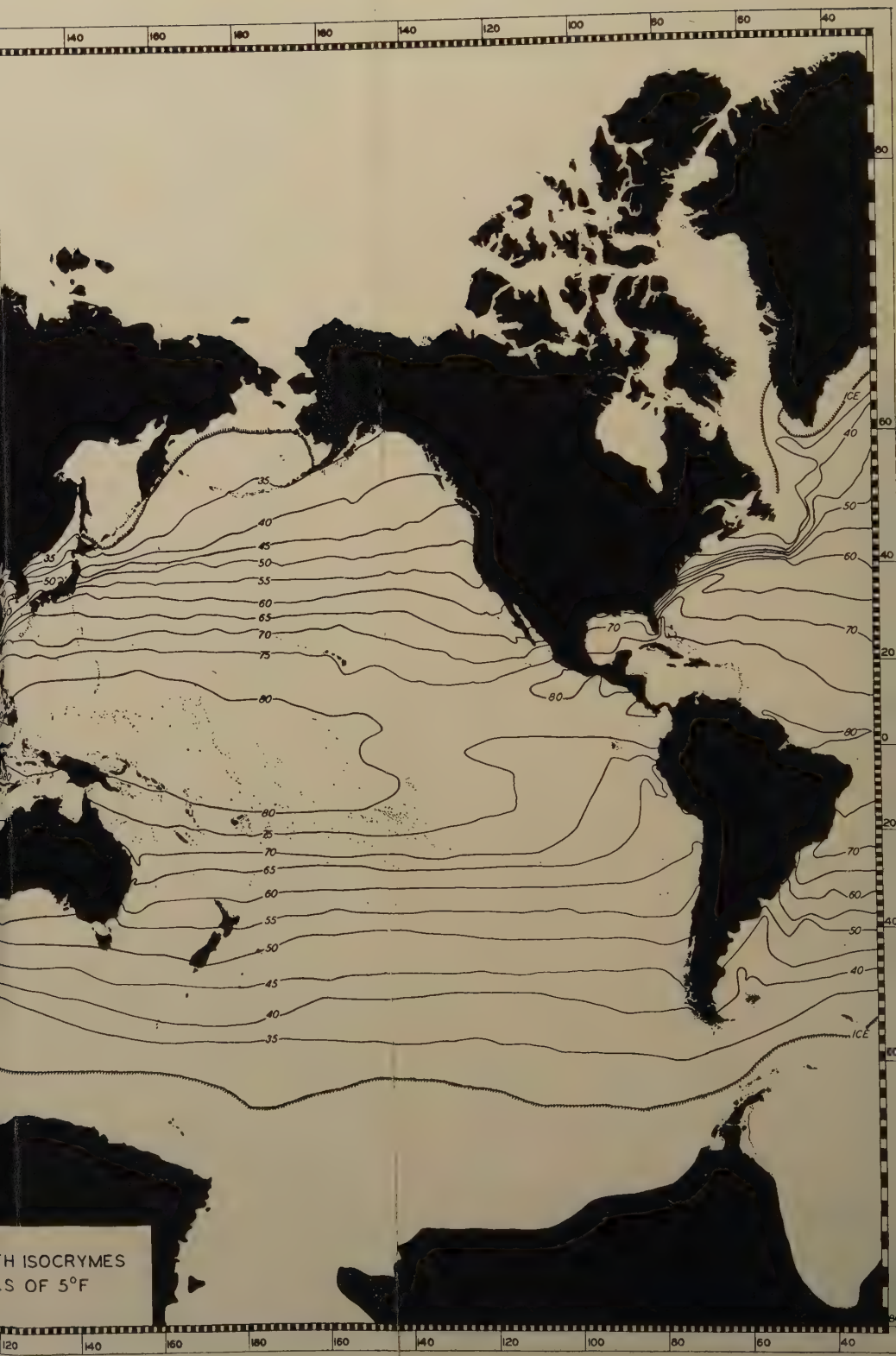
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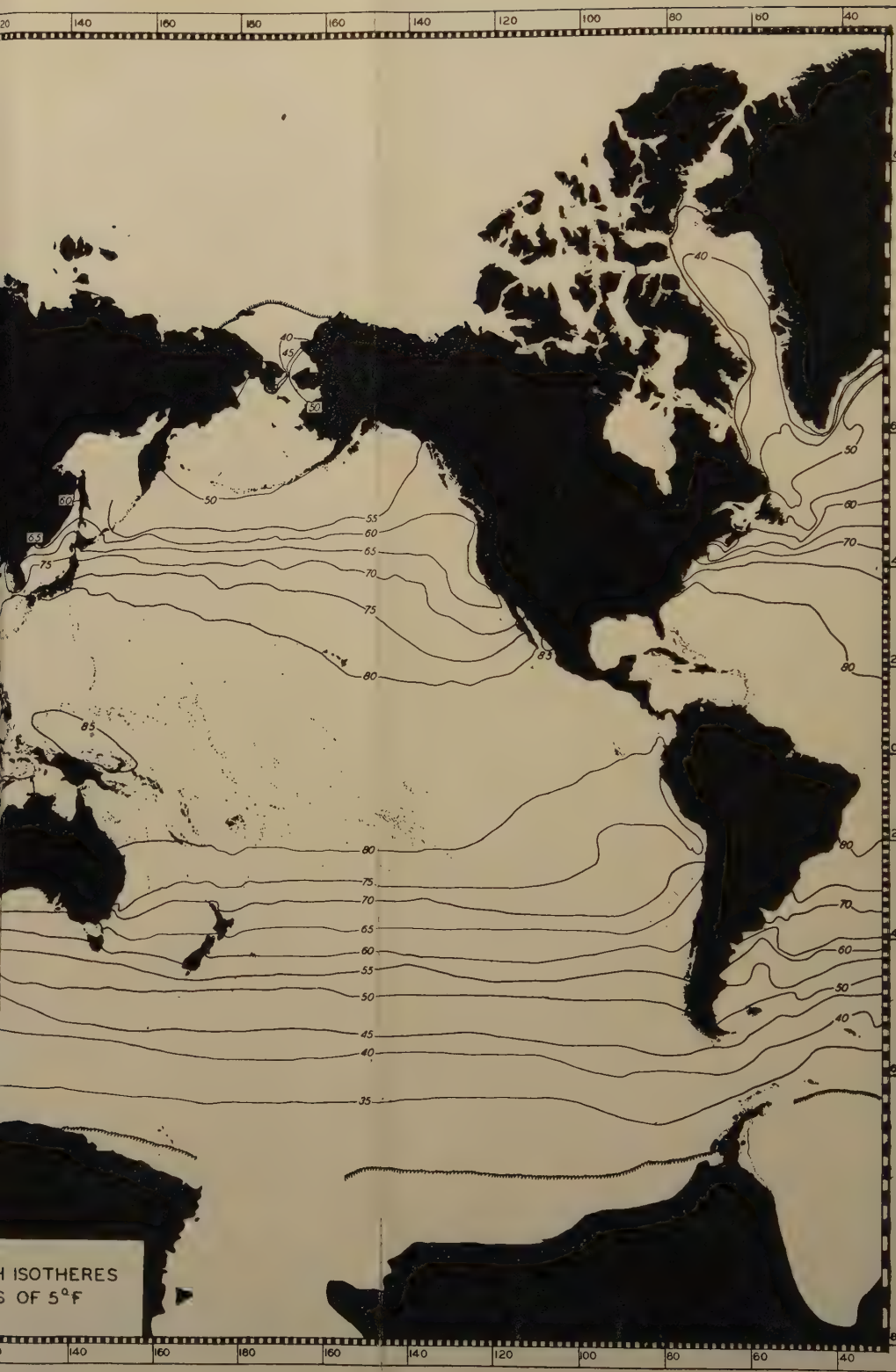
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